Introduction to Probability, Statistics and Data Handling	**Central Limit Theorem**
Tutorial 6	

Discussion:

Suppose X is a **random variable with a distribution that may be known or unknown** (it can be any distribution). X represents the population. We denote μ **as the mean of** $\boldsymbol{X}$ **and** $\boldsymbol{\sigma}$ **as the standard deviation of** $\boldsymbol{X}$.

Suppose that you draw **random samples of size** $\boldsymbol{n}$ and calculate the mean values of your sample. Name they as $\bar{x}$, so you have **n** values of $\bar{x}$, which **are sample means and** $\boldsymbol{\bar{X}}$ is also a random variable.

If n increases the random variable $\boldsymbol{\bar{X}}$ **tends to be normally distributed with a mean value** $\boldsymbol{\mu}$ **and standard deviation** $\boldsymbol{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$: $\boldsymbol{\bar{X} \rightarrow N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)}$. It means that if you draw random sample of size n, the distribution od sample means has the normal distribution with the same mean as the population and a variance that is equal the population variance divided by the sample size n.

The standard variable Z is than defined as: $\boldsymbol{Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}$; $Z \rightarrow N(0,1)$

1. An unknown distribution has a mean of 90 and a standard deviation of 15. Samples of size $n = 25$ are drawn randomly from the population.
   a) Find the probability that the sample mean is between 85 and 92.
   b) Find the value that is two standard deviations above the expected value of the sample mean.
2. Cans of a cola beverage claim to contain 16 ounces. The amounts in a sample are measured and the statistics are n = 34, $\bar{x}$ = 16.01 ounces. If the cans are filled so that μ = 16.00 ounces (as labelled) and σ = 0.143 ounces, find the probability that a sample of 34 cans will have an average amount greater than 16.01 ounces. Do the results suggest that cans are filled with an amount greater than 16 ounces?
3. An unknown distribution has a mean of 90 and a standard deviation of 15. A sample of size 80 is drawn randomly from the population.
   a) Find the probability that the sum of the 80 values (or the total of the 80 values) is more than 7 500.
   b) Find the sum that is 1.5 standard deviations above the mean of the sums.
4. In a recent study, the mean age of tablet users is 35 years. Suppose the standard deviation is ten years. The sample size is 39.
   a) What are the mean and standard deviation for the sum of the ages of tablet users? What is the distribution?
   b) Find the probability that the sum of the ages is between 1,400 and 1,500 years.
   c) Find the 90th percentile for the sum of the 39 ages.
5. A study involving stress is conducted among the students on a college campus. The stress scores follow a uniform distribution with the lowest stress score equal to one and the highest equal to five. Using a sample of 75 students, find:
   a) The probability that the mean stress score for the 75 students is less than two.
   b) The 90th percentile for the mean stress score for the 75 students.
   c) The probability that the total of the 75 stress scores is less than 200.
   d) The 90th percentile for the total stress score for the 75 students.