## Random Variables and Probability Distribution

## Tutorial 3

1. Let $X$ denote the number of boys in a randomly selected three-child family. Assuming that boys and girls are equally likely, construct the probability distribution of $X$.
2. The probability distribution of the random variable $X=$ number of changes in major is shown below:

| $X=\#$ changes in major | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Probability | 0.135 | 0.271 | 0.271 | 0.180 | 0.090 | 0.036 | 0.012 | 0.003 | 0.002 |

a) What is the probability that a college student will change majors at most once?
b) John's parents are concerned that he has decided to change his major for the second time. John claims that he is not unusual. What is the probability that a randomly selected college student will change his major as often as or more often than John?
c) What is the probability that a student will change majors 5 or 6 times?
d) What is the probability that a student will change majors at least once?
e) How often would John need to change his major to be considered unusual?
3. You apply for a job in a factory that claims that the average salary is above $3,500 €$. With a sad surprise you see that your first earning is $2,000 €$. Do you have a strong proof that the ad was a fake? Construct a probability distribution for a random variable that describes salary for a staff that consist of 100 workers with $2,000 €, 10$ managers that earn $10,000 €$ and two directors with 50,000€.
4. A men's soccer team plays soccer zero, one, or two days a week. The probability that they play zero days is 0.2 , the probability that they play one day is 0.5 , and the probability that they play two days is 0.3 . Find the long-term average or expected value, $\mu$, of the number of days per week the men's soccer team plays soccer.
a. Construct the probability distribution of $X$
b. Compute the expected value $E(X)$ of $X$.
c. Compute the standard deviation $\sigma$ of $X$.
5. The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between zero and 15 minutes, inclusive.
a) What is the probability that a person waits fewer than 12.5 minutes?
b) On the average, how long must a person wait? Find the mean, $\mu$, and the standard deviation, $\sigma$.
c) Ninety percent of the time, the time a person must wait falls below what value?
6. Given the Random Variable X has density function:

$$
f(x)=\left\{\begin{array}{lc}
2 x & 0<x<a \\
0 & \text { otherwise }
\end{array}\right.
$$

a) Determine a
b) Find $\mathrm{P}\left(\frac{1}{2}<\mathrm{X}<\frac{3}{4}\right)$ and $\mathrm{P}\left(-\frac{1}{2}<\mathrm{X}<\frac{1}{2}\right)$.

