1.1. Weak Interaction in the Standard Model

The Standard Model (SM) is a quantum field theory (QFT) that describes the spin-1/2 particles (fermions – leptons and quarks), interactions between them via spin-0 bosons exchange (photon, gluons, $W^{\pm}$ and Z^0) and spin-1 Higgs boson responsible for the generation of particles' mass. In QFT physical particles are interpreted as excitation of underlying fields.

The mathematical structure of the SM symmetry is described by the product of the gauge groups $SU(3) \times SU(2) \times U(1)$. The first term pertains to the symmetry of strong interaction, the two following represent the electromagnetic and weak interactions. The number of intermediate bosons corresponds to the number of respective group generators (eight in $SU(3)$ - gluons, in $SU(2)$ – three weak bosons, and one in $U(1)$ - photon). Due to gauge invariance symmetry of the Lagrangian, the bosons are massless particles what is hard to reconcile with current experiments. In addition, after the discovery of the massive weak bosons, it was also observed that $W^{\pm}$ violate parity, only interacting with left-handed particles, whereas the Z^0 boson interacts with both right and left-handed particles. These experimental evidences showed that the description of weak interaction and the change of quarks' flavour must internally include breaking of the parity symmetry and explain the mass of weak bosons.

In general, the Lagrangian might be decomposed into term describing separately free-particle $\mathcal{L}_f$ and interactions $\mathcal{L}_I$, in the simplest way denoted as:

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I. \tag{5.1}$$

These two terms must be written in such a way that the Lagrangian is gauge invariant and describe the observed, physical states.

The free fermion of mass m is described by the Dirac Lagrangian:

$$\mathcal{L}_D = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi. \tag{5.2}$$

Here ψ is a Dirac spinor that describes a fermion, and γ^{μ} are the Dirac matrices. The Euler–Lagrange equation for Dirac Lagrangian is denoted as the *Dirac equation*:

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0 \tag{5.3}$$

which has four solutions describing the spin-half particles and antiparticles (this interpretation is called *Dirac representation*).

Following the experimental evidences, the charged currents (CC) of weak interaction distinguish between the left- and right-handed chiral components of the Dirac spinors, $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma^5)\psi$, thus the Eq. 5.2 can be rewritten as (in so-called *chiral representation*):

$$\mathcal{L}_{ch} = \bar{\psi}_R i\gamma^{\mu}\partial_{\mu}\psi_R + \bar{\psi}_L i\gamma^{\mu}\partial_{\mu}\psi_L - m[\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R], \tag{5.4}$$

what shows that the kinetic part does not mix the left- and right- components of the spinor (the left- and right-components interact independently), while the mass term involves couplings of the left- and right-handed components. It is obvious that the Lagrangian of the form as in Eq. (5.4) is not gauge invariant.

The weakly interacting particles are grouped into multiplets; the left-handed SU(2) isospin doublets of leptons L_L^i and quarks Q_L^i:

$$L_L^1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \qquad L_L^2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \qquad L_L^3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L,$$

$$Q_L^1 = \begin{pmatrix} u \\ d \end{pmatrix}_L, \qquad Q_L^2 = \begin{pmatrix} c \\ s \end{pmatrix}_L, \qquad Q_L^3 = \begin{pmatrix} t \\ b \end{pmatrix}_L,$$

and the right-handed chiral charged weak fermion iso-singlets (index $i = 1,2,3$ stands for three quarks and leptons generation, U_R^i and D_R^i denote the "up" and "down" from i-th family quarks respectively, L_R^i describes three generations of right chiral leptons) :

$$U_R^i = (u \quad c \quad t)_R, \qquad D_R^i = (d \quad s \quad b)_R, \qquad L_R^i = (e \quad \mu \quad \tau)_R.$$

In this approach, the neutrinos are taken to be massless, therefore left-handed, so (in agreement with the observation) there are no right-handed neutrino singlets.

The electroweak Lagrangian for leptons is given by:

$$\mathcal{L}_{EW} = \sum_{i=1,2,3} \bar{L}_L^i \gamma^{\mu} D_{\mu} L_L^i + \sum_{i=1,2,3} \bar{L}_R^i \gamma^{\mu} D_{\mu} L_R^i - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu} \vec{W}^{\mu\nu}, \tag{5.5}$$

where the covariant derivate D_μ is introduced: $D_\mu \equiv i\partial_\mu - \frac{g}{2}\vec{\tau}\cdot\overrightarrow{W_\mu} - \frac{g'}{2}YB_\mu$. Here $\vec{\tau}$ are the Pauli matrices, g and g' are coupling strength to the $\overrightarrow{W_\mu}$ and B_μ fields (which are massless bosons in gauge theory), Y is the weak hypercharge. The Eq. 5.5 shows that the left-handed fermions interact with both the $\overrightarrow{W_\mu}$ and B_μ fields, whereas the right-handed fermions only with B_μ field, so that the $\frac{g}{2}\vec{\tau}\cdot\overrightarrow{W_\mu}$ is zero for the right-handed components[1].

In the Standard Model, the fields B_μ and $\overrightarrow{W_\mu} = (W_1, W_2, W_3)$ are attributed to the massless photon and the massive $W^\pm$ and Z^0 bosons respectively. In order to make the massless fields in the Eq. 5.5 physical and to regain gauge invariance in the Lagrangian, an additional doublet of scalar fields, called Higgs field, is added: $\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$.
The Higgs field is added to the Lagrangian in the form of:

$$\mathcal{L}_{Higgs} = \left(D_\mu\phi\right)^\dagger\left(D_\mu\phi\right) - V(\phi) + \mathcal{L}_Y, \tag{5.6}$$

where the potential $V(\phi) = \mu^2\phi^\dagger\phi - \frac{\lambda}{4}(\phi^\dagger\phi)^4$, μ and λ are constants, and $\mathcal{L}_Y$ is a Yukawa interaction term. If $\mu^2 < 0$, the value of minimal $|\phi|$ (referred as the vacuum expectation value υ) is non-zero and the symmetry is spontaneously broken. This gives rise to the mass of fields B_μ and $\overrightarrow{W_\mu}$ which gain physical properties in that way **Błąd! Nie można odnaleźć źródła odwołania.**.

By adding the Yukawa term of the form of $\mathcal{L}_Y = -g\bar{\psi}_L\phi\psi_R + h.c.$, the Higgs field give mass also to the fermions ($h.c.$ stands for *hermitian conjugate*):

$$\mathcal{L}_Y = Y_{ij}^d\, \bar{Q}_L^i\, \phi\, D_R^j + Y_{ij}^u\, \bar{Q}_L^i \phi\, U_R^j + Y_{ij}^l\, \bar{L}_L^i\, \phi\, L_R^j + h.c. \tag{5.7}$$

The matrices $Y_{ij}^d, Y_{ij}^u, Y_{ij}^l$ are complex matrices that represent the couplings between different families (generations) ij of quarks up-type, $d$own-type and leptons l. The Yukawa part of the Lagrangian is invariant under the *CP* symmetry if $Y_{ij} = Y_{ij}^*$.

After spontaneous symmetry breaking the mass terms for fermions are proportional to the Y_{ij}^k matrices:

$$M_{ij}^d = \upsilon\frac{Y_{ij}^d}{\sqrt{2}}, \qquad M_{ij}^u = \upsilon\frac{Y_{ij}^u}{\sqrt{2}}, \qquad M_{ij}^l = \upsilon\frac{Y_{ij}^l}{\sqrt{2}}.$$

The matrices $M_{ij}^{d,u,l}$ can have off-diagonal elements, allowing the mixing between quark and lepton families, d and u represent the down and up families of quark (d, s, b) and (u, c, t) respectively, whereas l stands for lepton families **Błąd! Nie można odnaleźć źródła odwołania.**.
In order to diagonalize the quark mass matrices one can define a unitary transformation V:

$$M_{diag}^d = V_{dL}M^dV_{dR}^\dagger$$

$$M_{diag}^u = V_{uL}M^uV_{uR}^\dagger.$$

This allows to introduce the Cabbibo-Kobayashi-Maskawa (CKM) matrix **Błąd! Nie można odnaleźć źródła odwołania.**:

$$V_{CKM} = \left(V_{uL}V_{dL}^\dagger\right)_{ij}. \tag{5.8}$$

By convention the weak (interaction) and mass eigenstates or the quarks are chosen such that they are equal for up-type quarks, while the down-type quarks are rotated using the CKM matrix:

$$\begin{pmatrix}d'\\ s'\\ b'\end{pmatrix} = V_{CKM}\begin{pmatrix}d\\ s\\ b\end{pmatrix}, \qquad V_{CKM} \equiv \begin{pmatrix}V_{ud} & V_{uc} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}, \tag{5.9}$$

where each element V_{ij} expresses the coupling strength of the weak interaction between the quarks i and j and transforms (rotate) the quarks from the physical mass (flavour) eigenstates (d, s, b) into weak eigenstates (d', s', b').

In terms of the mass eigenstates (u, c, t) and (d, s, b) the Lagrangian takes the form:

$$\mathcal{L}_{int} = -\frac{g}{\sqrt{2}}\left[(\bar{u}, \bar{c}, \bar{t})_L\,\gamma^\mu\, V_{CKM}\begin{pmatrix}d\\ s\\ b\end{pmatrix}_L W_\mu^- + \left(\bar{d}, \bar{s}, \bar{b}\right)_L\gamma^\mu\, V_{CKM}^\dagger\begin{pmatrix}u\\ c\\ t\end{pmatrix}_L W_\mu^+\right] \tag{5.10}$$

[1] The effect of $\overrightarrow{W_\mu}$ on fermion fields is to change between the lower and upper components of the doublets, so is effectively zero for right-handed spinor. It means that the weak interaction maximally violates P parity.

To summarise: the simple shape of the Lagrangian of weak interactions in Eq. 5.1 must have been rewritten in order to describe the physical, observed states and to maintain the gauge invariance. After the spontaneous symmetry breaking it gains the form that describes separately the fermion field $\mathcal{L}_f$, Higgs field $\mathcal{L}_H$ and gauge bosons fields $\mathcal{L}_G$:

$$\mathcal{L}_{EW} = \mathcal{L}_f + \mathcal{L}_H + \mathcal{L}_G. \tag{5.11}$$

The fermion component pertains to the mixed states of quarks and contains the terms for free particle and interaction between fermions and Higgs field proportional to the fermions' mass, interactions through charge currents with $W^{\pm}$ exchange and neutral currents with Z^0 and the electromagnetic interaction with γ exchange. The Higgs term generates the mass of the bosons $W^{\pm}$ and Z^0, which enter the Lagrangian as massless gauge bosons in the last term of Eq. 5.11.

The QFT of the SM, after the spontaneous symmetry breaking, is described by the product of the gauge groups $SU(3)_C \times SU(2)_L \times U(1)_Y$. The first term pertains to the colour (C) symmetry of strong interaction, the two following represent the electroweak theory: the interaction of left chiral states by the weak charge currents and neutral currents that contains the interaction with mixed Z^0 and γ fields. The mathematical structure of the Lagrangian given by the Eq. 5.11 internally includes the possibility of *CP* symmetry violation – thanks to the different mass of quarks and quark mixing and because of the irreducible complex phases in V_{CKM}. The Lagrangian for the weak interaction features 17 independent parameters that must be determined experimentally.

1.2. CP Violation in the Standard Model

The two terms in the square brackets in Eq. (5.10) transform into each other under *CP* symmetry, V_{CKM} is the unitary Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix containing the strength of the flavour-changing weak decays in three quark generations. As the CKM matrix is a 3×3 complex matrix there are in general 18 free parameters. The unitarity conditions $V_{CKM}^{\dagger} V_{CKM} = V_{CKM} V_{CKM}^{\dagger} = \mathbb{I}$ reduce this to nine. Each quark field can also be multiplied by a phase factor with no change in the Lagrangian. This reduces the CKM matrix to have four non-reducible parameters, which are three magnitudes of quark transition and a single phase (this parametrisation is regarded as *standard* parametrisation).

Another practical and commonly used parametrisation of the CKM matrix is the Wolfenstein parametrisation **Błąd! Nie można odnaleźć źródła odwołania.**. It clearly reveals the hierarchy (relative strength) of the quark transitions since each element is proportional to the parameter $\lambda = \sin\theta_C \approx 0.22$:

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda - A^2\lambda^5\left(\rho + i\eta - \frac{1}{2}\right) & 1-\frac{\lambda^2}{2}-\left(\frac{1}{8}+\frac{A}{2}\right)\lambda^4 & A\lambda^2 \\ A\lambda^3\left[1-(\rho+i\eta)\left(1-\frac{\lambda^2}{2}\right)\right] & -A\lambda^2 - A\lambda^4\left(\rho+i\eta-\frac{1}{2}\right) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^7), \tag{5.12}$$

where $A,\ \rho,\ \eta, \sin\theta_C$ are real parameters[2], $\sin\theta_C$ being the Cabbibo mixing angle for two quark generations. The η parameter was introduced to account for *CP* violation in the SM. The above representation contains the terms proportional to λ^6, which are necesairly to study the $B_S^0 - \bar{B}_S^0$ mixing and can be studied thanks to the current precision of the LHCb measurements.

If *CP* symmetry was conserved, the CKM matrix would fulfil the condition: $V_{CKM} = V_{CKM}^*$ and all elements would be real. Therefore, the non-zero imaginary part of the V_{CKM} elements is necessary to describe *CP* violation in the SM. The existence of three quark generations was predicted by Kabayashi and Maskawa as an explanation of *CP* violation in weak quark transitions. For two quark families, there are no phase terms in the quark mixing matrix and no complex Yukawa couplings.

In the SM the magnitude of *CP* violation is determined by the *Jarlskog parameter* J_{CP}, defined as the imaginary part of the products of the CKM matrix elements, directly related to the η parameter **Błąd! Nie można odnaleźć źródła odwołania.**:

[2] The current values for CKM parameters are: $A = 0.814^{+0.021}_{-0.022}$, $\rho = 0.135^{+0.031}_{-0.016}$, $\eta = 0.349^{+0.015}_{-0.017}$, $\lambda = 0.2257^{+0.0009}_{-0.0010}$ **Błąd! Nie można odnaleźć źródła odwołania.**.

$$J_{CP} = \left|\Im\left(V_{i\alpha}V_{j\beta}V_{i\beta}^{*}V_{j\alpha}^{*}\right)\right| = \lambda^6 A^2 \eta \left(1 - \frac{\lambda^2}{2}\right) + \sigma(\lambda^{10}) \sim 10^{-5}, \tag{5.13}$$

what shows that *CP* symmetry is violated if $J_{CP} \neq 0$ and the violation effects are small in the SM. Therefore, the matter dominance over antimatter in current Universe is hardly explained by the *CP* violation exclusively in electroweak decays what enhance the quest for new sources of *CP* violation that could come from the physics Beyond the SM (BSM).

The CKM matrix is unitary so we can formulate 12 orthogonality conditions for the complex elements: $\sum_k V_{ki}V_{kj}^{*} = \delta_{ij}$. Since the complex numbers can be represented by vectors, six independent equations form the triangles in a complex plane, all having the same area $\frac{J_{CP}}{2}$. In case *CP* was conserved, the triangles would be flat. All but two triangles are very different in shape. These two, because of comparable lengths of their respective sides (proportional to λ^3), can be studied experimentally. They are obtained by multiplying the first and the third column of the CKM matrix and the first and the third row. These conditions involve the *up* and *beauty* quarks so pertain to the B^0 system:

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0, \tag{5.14a}$$

$$V_{ud}V_{td}^{*} + V_{us}V_{ts}^{*} + V_{ub}V_{tb}^{*} = 0. \tag{5.14b}$$

The third condition refers to the B_s^0 meson system:

$$V_{us}V_{ub}^{*} + V_{cs}V_{cb}^{*} + V_{ts}V_{tb}^{*} = 0. \tag{5.15}$$

These conditions are represented by so-called *unitary triangles*. When the condition in Eq. 5.14a is divided by the factor $V_{cd}V_{cb}^{*}$, it gives the relationship:

$$\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}} + 1 + \frac{V_{td}V_{tb}^{*}}{V_{cd}V_{cb}^{*}} = 0 \tag{5.16}$$

and is called as *the Unitary Triangle* (UT), schematically drawn in Fig. 5.1a. The apex of the UT is denoted as: $(\bar{\rho}, \bar{\eta}) \equiv \left[\rho\left(1 - \frac{1}{2}\lambda^2\right), \eta\left(1 - \frac{1}{2}\lambda^2\right)\right]$, while the other UT apexes are (0,0) and (1,0). Three internal angles, known as *weak phases* are defined that way as:

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\right), \qquad \beta = \arg\left(-\frac{V_{cd}V_{cb}^{*}}{V_{td}V_{tb}^{*}}\right), \qquad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right). \tag{5.17}$$

When the Eq. 5.14b is rescaled by $V_{us}V_{cb}^{*}$, the second nonsquashed triangle is constructed; its apex is located at the point (ρ, η) and is tilted with respect to the real axis by a small angle $\delta\gamma = \arg\left(-\frac{V_{ts}V_{tb}^{*}}{V_{cs}V_{cb}^{*}}\right)$, see Fig. 5.1b.

Neutral mesons B_s^0, D^0, K^0 systems are represented by the other squashed triangles with the same area $\frac{J_{CP}}{2}$. Eq. 5.15 refers to the B_s^0 system and defines the *CP* violating phase β_s:

$$\beta_s = \arg\left(-\frac{V_{ts}V_{tb}^{*}}{V_{cs}V_{cb}^{*}}\right), \tag{5.18}$$

which is equivalent to the $\delta\gamma$, so $\delta\gamma \equiv \beta_s$. This triangle has lengths of sides that differ by orders of magnitude, so one angle, β_s, is supposed to be very small, $\beta_s \cong \lambda^2\eta$.

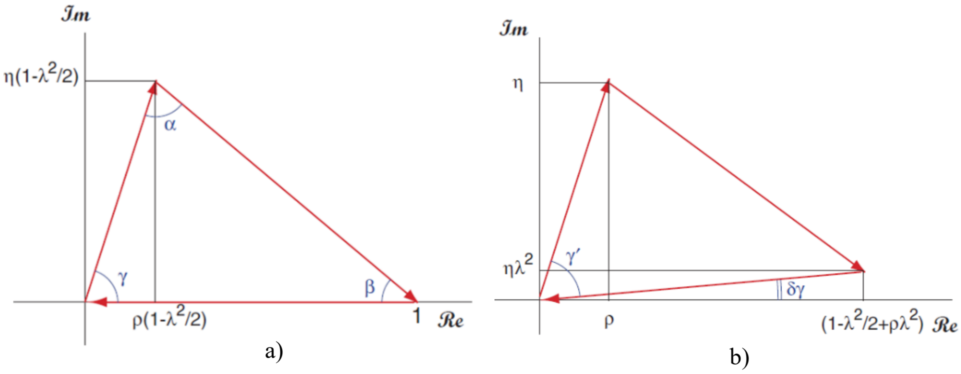

Fig. 5.1. a) The Unitary Tringle of the B^0 system from Eq. 5.14a divided by $V_{cd}V_{cb}^*$. b) The second triangle represents Eq. 5.14b divided by $V_{us}V_{cb}^*$.

There are no predictions of Yukawa couplings from the SM, all CKM elements must be determined experimentally. From the experimental point of view, it is practical to express the CKM matrix in the *standard* representation, comprising of respective coupling constants, exposing the parameters to determine: magnitudes of amplitudes and weak phases:

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} \tag{5.19}$$

The current constrains imposed on the CKM parameters are shown in Fig. 5.2. The results are combinations of many experiments, like B-factories (Belle, BaBar), CDF and LHC experiments (LHCb, CMS, ATLAS). The

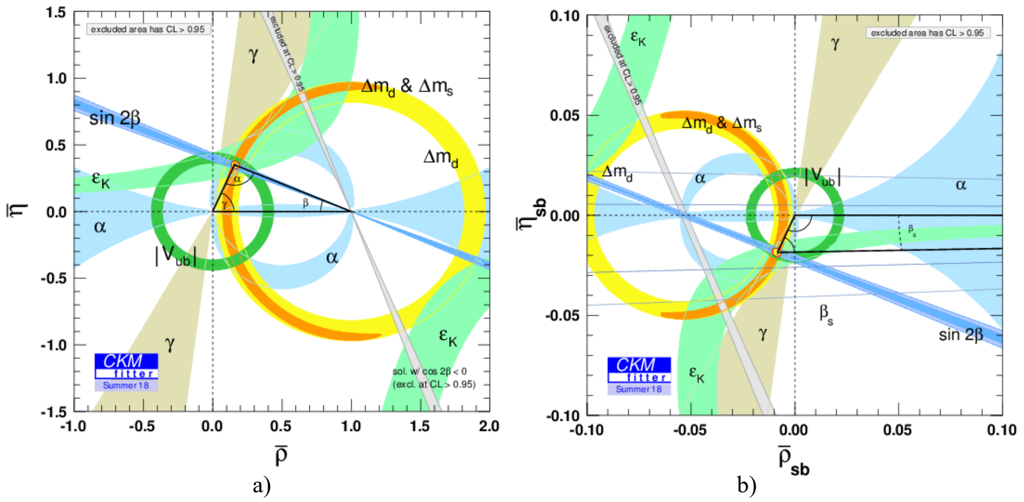

Fig. 5.2. a) Experimental constraints of The Unitary Triangle in the $(\bar{\rho},\bar{\eta})$ plane. b) Measurements of the β_s phase in the $(\bar{\rho}_{sb},\bar{\eta}_{sb})$ plane defined as: $\bar{\rho}_{sb} + i\bar{\eta}_{sb} = -\frac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*}$ **Błąd! Nie można odnaleźć źródła odwołania.**

triangles are constrained by measurement of their sides and angles, overconstraining helps in spotting the tensions between the measured parameters. The determination of the CKM matrix parameters is regarded as a fundamental test of the unitarity condition, any discrepancy would indicate the phenomena of the physics beyond the Standard Model. The precision of the measurements of the CKM parameters has increased more than of order of magnitude since the first published result and no clear evidence of physics BSM have been found **Błąd! Nie można odnaleźć źródła odwołania.**.

The brief description of the CKM angle measurements in presented in Table 5.1. The current measurements are summarised in the matrix (5.20) **Błąd! Nie można odnaleźć źródła odwołania.**.

Table 5.1. Selected measurements constraining the CKM matrix **Błąd! Nie można odnaleźć źródła odwołania.**.

Phase	Value	Method of measurement
$\boldsymbol{\alpha}$	$\alpha = (84.5^{+5.9}_{-5.2})°$	Time-dependent CP asymmetries in $B^{0+} \to hh$. Penguin amplitudes need be considered.
$\boldsymbol{\beta}$	$sin2\beta = 0.691 \pm 0.017$ $\beta = (21.9 \pm 0.7)°$	Dominated by B^0 decays to charmonium and $K^0_{S,L}$ states.
$\boldsymbol{\gamma}$	$\gamma = (73.5^{+4.2}_{-5.1})°$	The tree-level-only processes. Comparison with indirect (loop) processes shows a tension in results.
$\boldsymbol{\beta_s}$	$-2\beta_s = 0.021 \pm 0.31$	Time-dependent CP asymmetry in $B^0_s \to J/\psi\,\phi$ decays.

$$V_{CKM} = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359 \pm 0.00011 & 0.04214 \pm 0.00076 \\ 0.00896 \pm 0.00024 & 0.04133 \pm 0.00074 & 0.999105 \pm 0.00032 \end{pmatrix} \tag{5.20}$$