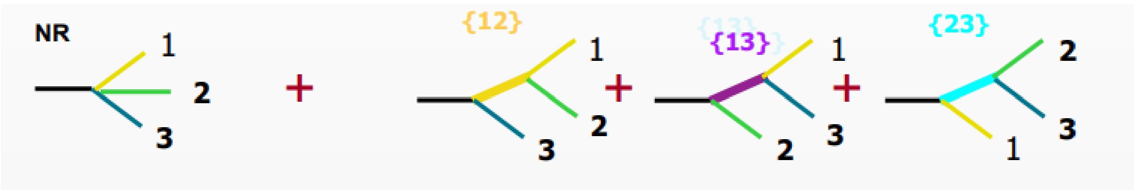
What are Dalitz Plots? lhcb/opendata-project, B. Lindquist SASS talk, 2010

A Dalitz plot is a visual representation of the phase-space of a three-body decay. It is named after its inventor, Richard Dalitz (1925–2006).

If there are no intermediate resonances in the three-body decay that is being analysed the plot is equally populated with events inside its physical boundaries.
If there are intermediate resonances they show up as bands on the plot with a higher density of events. Let us take the case where $B^+ \to K^+R^0$, where R^0 is a neutral particle resonance which can decay $R^0 \to K^+K^-$. We would observe an enhanced band of events across the Dalitz plot at a mass corresponding to that of the particle R^0. Particles also have a quantum mechanical property called spin. If R^0 has spin 0 then we will observe a continuous band of events across the plot. If R^0 has spin 1 then the band will have a minima, for spin 2 particles it will have two minima. Where resonances overlap that can interfere with each other, a property analogous to interference in light that you may be familiar with.



What are resonances?

Consider a wave function of a decaying particle:

$$\Psi(t) = \Psi(0)\mathrm{e}^{-iE_Rt}\ \mathrm{e}^{-t/2\tau} = \Psi(0)\ \mathrm{e}^{-t(iE_R+\Gamma/2)},\ \tau = 1/\Gamma$$

Particle may propagate in time or decay. The probability of finding it after the time t is:

$$I(t) = \Psi^*\Psi = \Psi(0)^2\mathrm{e}^{-t/\tau}$$

To find an energy distribution one must find the Fourier transformation of $\Psi(t)$:

$$\Psi(E) = \int \Psi(t)\mathrm{e}^{iEt}dt = \Psi(0)\int \mathrm{e}^{-t\left[\frac{\Gamma}{2}+iE_R-iE\right]}dt =$$

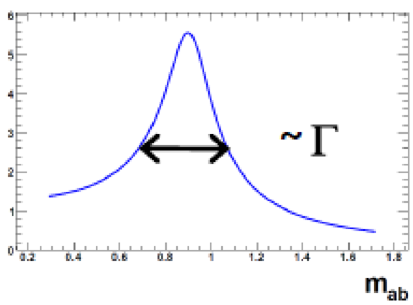
$$= \frac{K}{(E_R - E) - i\Gamma/2}$$

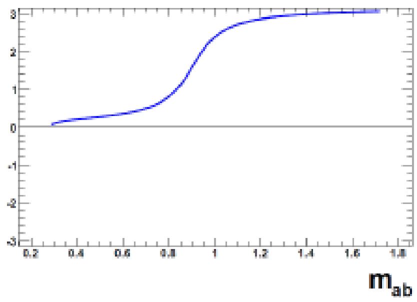
$$\sigma(E) = \Psi(E)\Psi^*(E) = \sigma_{max}\frac{\Gamma^2/4}{(E - E_R)^2 - \Gamma^2/4}$$

Magnitude

$$\boxed{A = |A|e^{i\phi}}$$

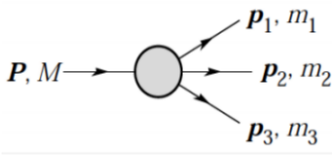
Phase

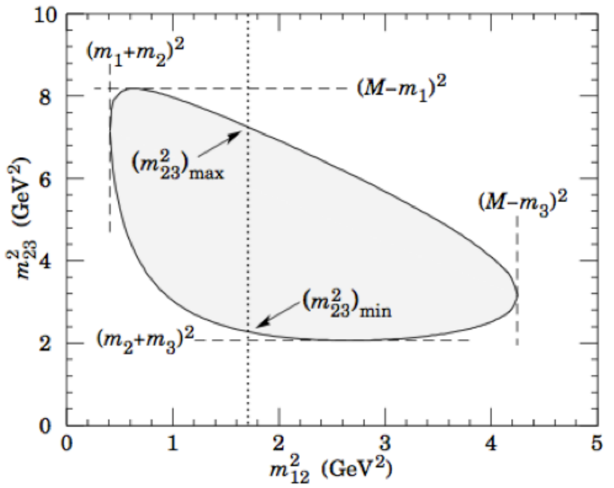




How do we construct a Dalitz Plot?

On each of the two axes of the Dalitz plot we plot a quantity with units of mass squared. Each of these masses correspond to the masses of a quasi-particle (R^0 above). It is the invariant mass of two final state particles combined. If we call the final state particles 1,2,3 these masses can be written m_{12}, m_{23}. The square of these masses is the mass plotted on the axes, as in this diagram (image credit: particle data group).





$$(m_i^2 + m_j)^2 \leq m_{ij}^2 \leq (M - m_k)^2$$

These masses are constrained in the values that they can take, there are maximal and minimal possible masses that define the boundaries of the plot since energy and momentum are conserved. The limits of the plot are shown in the diagram, where M is the mass of the original particle (for us the B meson) that decayed into the three final state particles (for us the kaons).

In the rest frame of M meson, the momenta of the daughters will lie in a plane. In a two-body decay, if the masses of parent and daughters are known, the magnitudes of the momenta of the daughters in the parent rest frame are fixed. In a three body decay, there are two additional degrees of freedom. The daughter particles can be paired in three possible ways, and the invariant masses of the pairs are: m_{12}, m_{13}, m_{23} . For example: $m_{12}^2 = (P_1 + P_2)^2 = (P_M - P_3)^2$, where P_i stands for four-momentum of i-th particle.

Consider a generic decay of a pseudo-scalar meson at rest, with mass M and four-momentum $P^\mu = (M,0)$, to three particles with masses m_i, four momenta $p^\mu_i = (E_i, \mathbf{p}_i)$ and energies E_i, where $i = 1, 2, 3$. Defining Lorentz invariant masses

$$\begin{aligned} m_{ij}^2 &= (p_i^\mu + p_j^\mu)^2 = m_i^2 + m_j^2 + 2E_iE_j - 2\vec{p}_i \cdot \vec{p}_j \\ &= (P^\mu - p_k^\mu)^2 = M^2 + m_k^2 - 2ME_k \end{aligned}$$

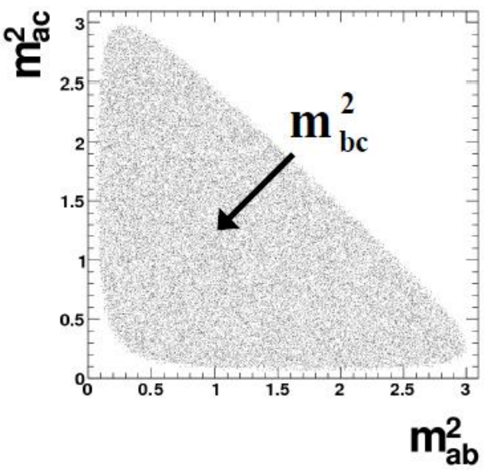
where the relation

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = M^2 + m_1^2 + m_2^2 + m_3^2$$

constrains the system to two independent values of m^2_{ij}

Any two of these are enough to remove the remaining degrees of freedom and to completely determine the decay kinematics in the parent particle rest frame. For example, the four-momentum of particle 3 is maximal when $m_{12} = m_1 + m_2$. The scatterplot formed by plotting m_{ij}^2 against m_{ik}^2 referred to as a Dalitz plot. Energy conservation means that all of the points in this plot should lie within a well-defined kinematic limit.

No resonances – phase-shape on Dalitz distribution



$$x = m^2_{ab} \qquad y = m^2_{ac}$$

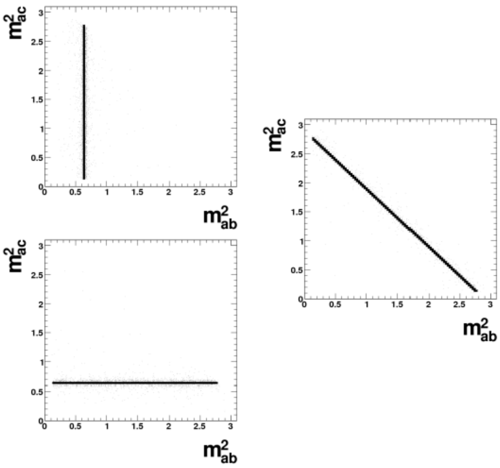
$$m^2_{ab} = (p^\mu_a + p^\mu_b)^2$$

Resonances on Dalitz plot

- E and p conservation imply that if r→ab, then:

  $$m^2_{ab} = m^2_r$$

- Resonances show up as bands on Dalitz plot.

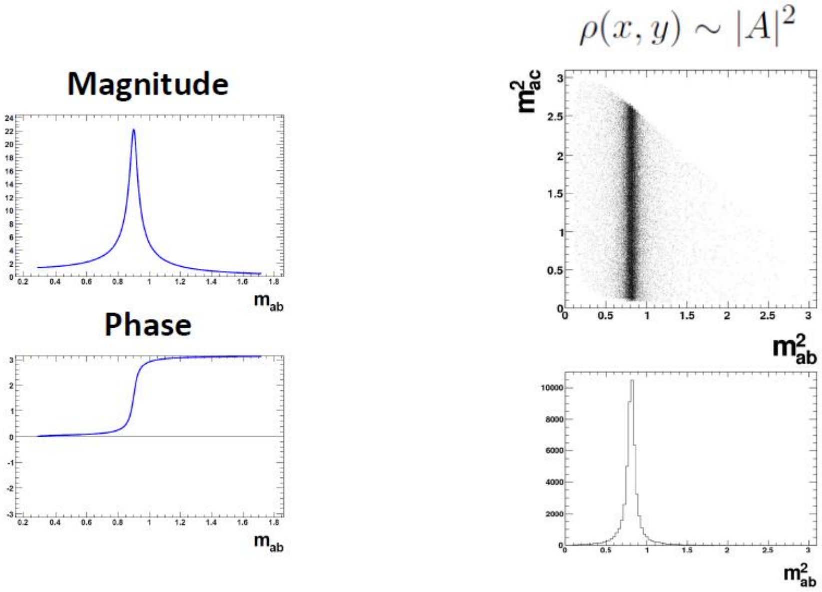


The density of points in a Dalitz plot, parameterised by the partial decay rate dΓ of a particle of mass M, is related to the spin-averaged matrix element for the decay M into an element of the Dalitz plot $dm_{12}^2 dm_{13}^2$ by:

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{8M}|M|^2 dm_{12}^2 dm_{13}^2.$$

Therefore non-uniformities in a Dalitz plot provide invaluable insights into the nature of M. They suggest the presence of intermediate two-body resonances.

In general, final state interactions differ between resonances, meaning that their relative strong phases vary as the resonances interfere. If the Dalitz plot can be fitted satisfactorily with a simple parameterisation made up of a finite number of resonances, the relative strong phases of these resonances can be measured directly. Fitting the Dalitz plot for both a particle and an antiparticle separately also gives information about weak phases.

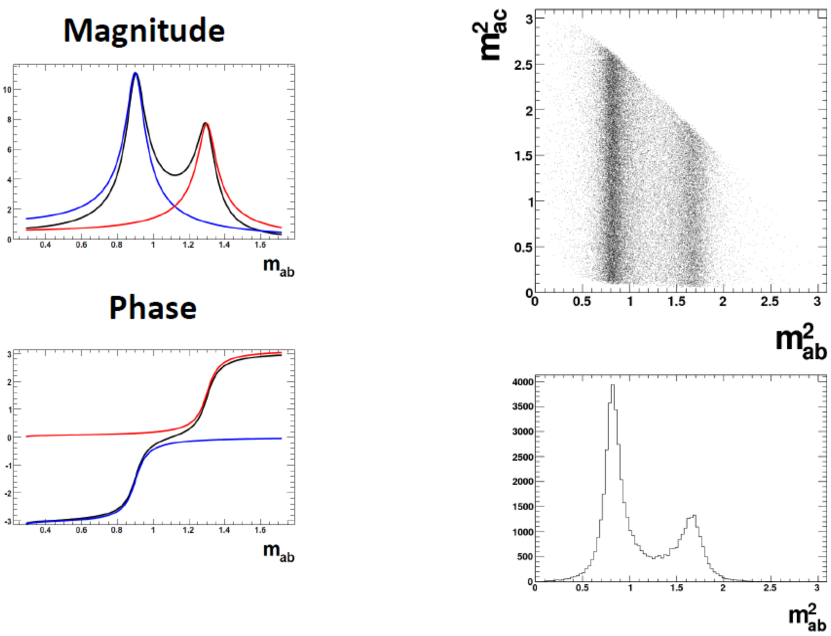


Brian Lindquist, Dalitz Plots SASS Talk

In so-called isobar model the overall matrix element for the three-body decay is a sum of the isobar model matrix elements with relative phases ϕ_i:

$$A = \sum_k c_k e^{i\phi_k} A_k$$

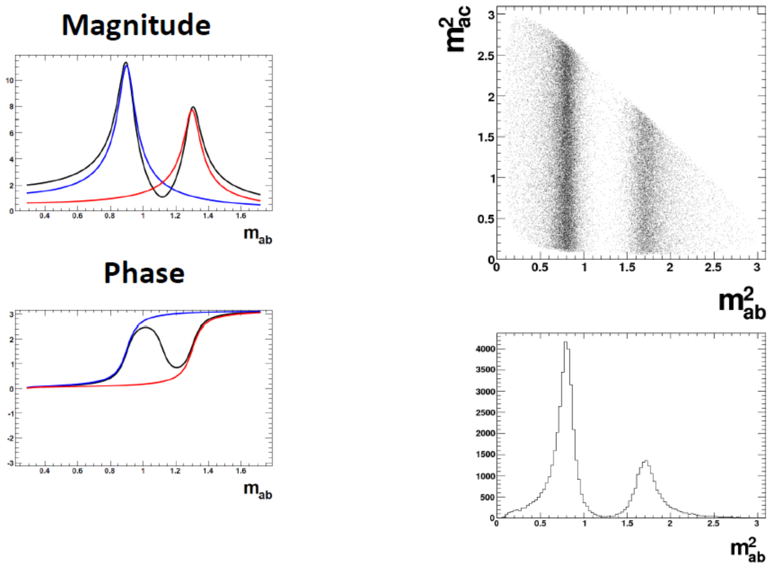
Constructive Interference



$A_k = A_k(m_{ab}^2, m_{ac}^2)$ are the Dalitz-plot dependent amplitudes (e.g. relativistic Breit-Wigner, or may be a "nonresonant" term).

c_k and ϕ_k are constants which can be measured in a maximum likelihood fit.

Destructive Interference

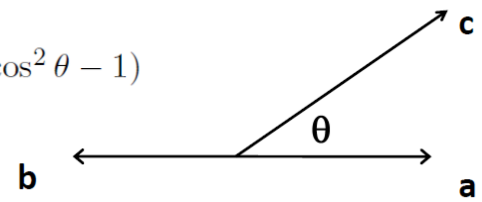


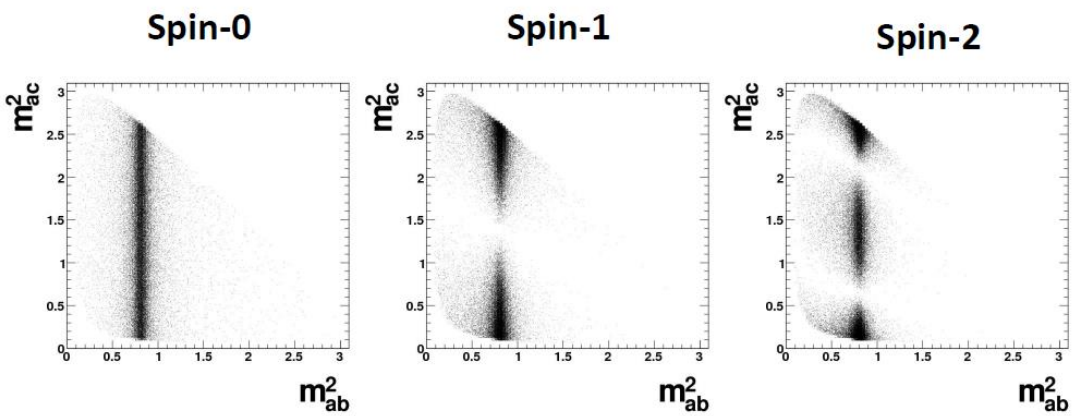
Resonance spins

If the resonance has spin S, and M, a, b, and c are spin-0, then decay amplitude is proportional to Legendre polynomial:

$$A \propto A_{RBW}(m_{ab})P_S(\cos\theta)$$

$$P_0(\cos\theta) = 1$$
$$P_1(\cos\theta) = \cos\theta$$
$$P_2(\cos\theta) = \tfrac{1}{2}(3\cos^2\theta - 1)$$





Dalitz plots and two body resonances [lhcb/opendata-project]

Aims:

- Produce Dalitz plots of the simulation and real data sample
- Create ordered and binned dalitz plots.
- Identify two body resonances in the Dalitz plots

In this stage we introduce you to an important technique for analysing decays of one particle (your charged B meson) into three bodies (the three kaons). This is known as a Dalitz plot.

The decay of the B meson can proceed either directly to the three-body final state or via an intermediate particle. For example, $B^+ \rightarrow K^+K^+K^-$, could proceed through the decay $B^+ \rightarrow K^+R^0$, where R^0 is a neutral particle resonance which can decay $R^0 \rightarrow K^+K^-$. Dalitz plots can be used to identify these resonances which are visible as bands on the Dalitz plot.

The kinematics of a three-body decay can be fully described using only two variables. The energies and momenta of the three kaons are not independent of each other as they all come from the decay of a B meson and energy and momentum are conserved. The axes of the plots conventionally are the squared invariant masses of two pairs of the decay products. It is a 2D plot, the x and y axes are both squared masses and the density of points in the plot shows the structure.

Consider our decay $B^+ \rightarrow K^+{}_1K^+{}_2K^-{}_3$, where we have numbered the kaons 1,2,3 to distinguish them. We can calculate the invariant mass of three possible combinations that could correspond to intermediate resonances $R^{++}{}_1 \rightarrow K^+{}_1K^+{}_2$, $R^0{}_2 \rightarrow K^+{}_1K^-{}_3$, and $R^0{}_3 \rightarrow K^+{}_3K^-{}_3$.

The potential $R^{++}{}_1$ would be a doubly charged resonance. We would not expect to see any resonances corresponding to this as mesons are composed of one quark and one anti-quark and their charges cannot add up to two units.

The potential $R^0{}_2$ and $R^0{}_3$ correspond to configurations in which we could see resonances. Hence you should compute the invariant mass combinations for these. The square of these masses should be used as the Dalitz variables.

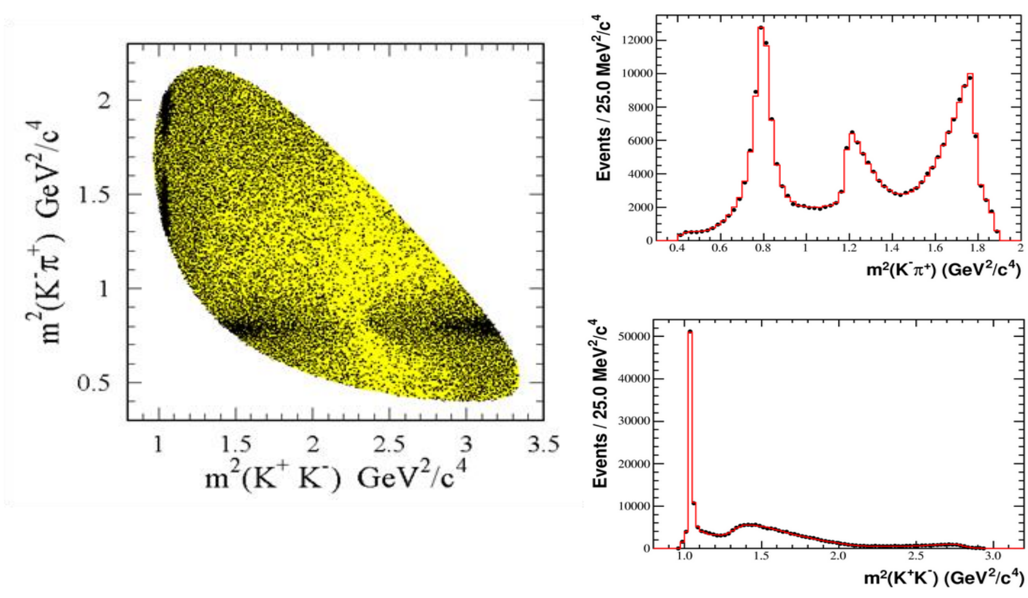
We suggest you make these plots first for the simulation data. In the simulation there are no intermediate resonances and your plot should be of uniform density inside the range physically allowed by energy and momentum conservation.

 # calculate the invariant masses for each possible hadron pair combination

 # plot the invariant mass for one of these combinations

 # make a Dalitz plot with labelled axes for the simulation data

B. Aubert et al., Phys. Rev. D83, 052001 (2011).:



Adding Dalitz plot for real data

Now draw a Dalitz plot for the real data. Check that the signs of the charge of the hadrons are correct to correspond to your potential neutral resonances R^0_2 and R^0_3.

 # calculate the invariant masses for each possible hadron pair combination in the real data

 # make a Dalitz plot for the real data (with your preselection cuts applied)

While drawing the Dalitz plot for the real data, label the axes accordingly. Compare the Dalitz plots of the real data with the one for the simulation. What are the most striking differences?

Ordering Dalitz variables

You can make a further improvement to allow you to observe the resonances easier. Your resonances R^0_2 and R^0_3 are both composed of the same particle types, K^+K^-, and hence have the same distributions. It is useful to impose an ordering which distinguishes the resonances. We can call the resonances R^0_{Low} and R^0_{High}. In each event R^0_{Low} is the resonance with the lower mass and the other corresponds to the higher mass combination of kaons. You can now use the mass of these ordered resonances as your Dalitz plot variables, thus effectively "folding" your Dalitz plot so that one axis always has a higher value than the other.

 # make a new Dalitz plot with a mass ordering of the axes

Binned Dalitz plot

You can improve the representation of your Dalitz plot by binning the data. The hist2d function can be used to make a 2D histogram. The number of bins specification in the hist2d function is the number of bins in one axis.

 # plot a binned Dalitz Plot

Two body resonances

You can now use your Dalitz plot to identify the intermediate resonances that you see in your plots. The resonances will have shown up as bands of higher density of points on the plots. You can use the particle data group tables of mesons to identify which particles these correspond to. The tables give the masses and widths of the particles and their decay modes. You are looking for mesons with the masses corresponding to where you see the bands and that decay into K^+K^-.

Congratulations! You have succesfully made a Dalitz plot and used it to observe the presence of intermediate particles in the decay of your charged B meson into three charged kaons.

Searching for local matter anti-matter differences

Aims:

- Observe matter antimatter differences (CP violation) in regions of the Dalitz plots of the B^+ and B^- mesons.
- For the data in these regions produce plots to best display the CP violation.

In a section above you searched for global CP violation. You probably did not find a result with very high significance.

CP violation may arise from interference between decays through different resonances, and hence the magnitude and sign of the CP violation may vary across the Dalitz plot. We can apply the same equation as in the global CP violation studybut apply this only to events in particular regions of the Dalitz plot.

$$A = \frac{(N^- - N^+)}{(N^- + N^+)}$$

Removing charm resonances

The analysis performed here is to study the CP violation in the charmless B meson decays to kaons. "charmless" means that the decay does not proceed through a charm quark. However, the most frequent decay of the B mesons occur through the *b* quark decaying into a *c* quark. The majority of these events can be removed by rejecting the events that are proceeding through a D^0meson (which contains the charm quark).

In the section above you plotted a histogram of the invariant mass of the intermediate resonances and will have observed the D^0meson in this and in the Dalitz plot. You should now reject events that are around the mass range of the D^0 meson to suppress this contribution. You can do this in your pre-selection on the data that you set-up earlier in the project.

This was also a simplification that we did not consider when we were calculating the global asymmetry. After you have applied this pre-selection your code will now recompute the global asymmetry with the D^0 meson contribution rejected. We have not yet observed CP violation in charm mesons and searching for this is another active area of current research.

Comparing Dalitz plots

Make separate Dalitz plots for the B^+ and the B^- decays. Local CP Violation will show up as an asymmetry between the B^+ and the B^-plots.

In order that the statistical error on the asymmetry in each bin is not over large the bins need to contain a reasonable number of entries. Hence you will probably need larger bins than when you were looking

for resonances in the section above. A suitable initial bin size might be .

 # make a Dalitz plot for the B+ events

 # make a Dalitz plot for the B- events

 # Make a plot showing the asymmetry between these two Daltz plots

 # i.e. calculate the asymmetry between each bin of the B+ and B- Dalitz plots and show the result in a other 2D plot

Observing a large asymmetry in some regions of the plot does not necessarily mean you have observed CP violation. If there are very few events in that region of the plot the uncertainty on that large asymmetry may be large. Hence, the value may still be compatible with zero.

You can calculate the statistical uncertainty on the asymmetry, for each bin of the plot, using the same formulas as you used in the global asymmetry section. You can then make a plot showing the uncertainty on the asymmetry.

Dividing the plot showing the asymmetry by the plot showing the statistical uncertainty you can then obtain the significance of the asymmetry in each bin. You can then plot the significance of the asymmetry to see if there is any evidence for CP violation.

 # Make a plot showing the uncertainty on the asymmetry

 # Make a plot showing the statistical significance of the asymmetry

Observing CP violation

From your studies of the asymmetry plot, and the plot of its significance, you will be able to identify regions in the Dalitz plots that show indications of sizeable and significant CP Violation. You may find you have several consecutive bins with significant positive, or negative, asymmetries. You may wish to try alternative binnings of the Dalitz plots to best isolate the regions in which the significant asymmetries occur.

You can select events that are in these regions of the Dalitz plots where you observe signs of CP Violation. You can then plot a simple 1D histogram of the invariant mass distribution of the B^+ and the B^- events, just as you did at the start of the project, but only for events that lie in the region of the Dalitz plot that you are interested in. Make the plots of the B^+ and the B^- events with the same scale, or superimpose the two plots, so that you can observe if the particle and anti-particle decay processes are occurring at the same rate.

 # Make a plot showing the invariant mass of the B+ meson particles

 # using events from a region of the Dalitz plot showing sizeable CP asymmetries

 # Make a plot showing the invariant mass of the B- meson particles using events from the same region

Congratulations! You should now have successfully observed significant evidence for CP Violation. You should have plots that clearly show that particle and anti-particle decay processes occur at different rates in local regions of the Dalitz plot. You may wish to compare your results with those published by the LHCb collaboration in this paper.

Well Done you have successfully completed your first particle physics analysis project. There are many more analyses that can be conducted with the data set that you have been provided and the skills that you have gained. Ideas for some of these are explored in the section below. Maybe you can discover something new!

Further analyses

The data set you have been provided is the full set of data recorded by LHCb preselected for decays of charged B mesons into three final state tracks. This data set has been used for two important publications, here and here.

We discuss here:

- Additional elements that you could add to your analysis of $B^+ \rightarrow K^+K^+K^-$
- Further analyses that you could perform with this data set

Adding extra sophistication

Systematic Uncertainties

In this analysis you considered the statistical uncertainty on the result. This occurs as a result of having only a limited number of events. In addition there are systematic uncertainties, these arise from biases in your measurement. Here we discuss three sources of these for this analysis.

- Production asymmetry. The LHC is a proton-proton collider and hence the initial state of the collision is not matter antimatter symmetric. Consequently B^+ and B^- mesons may not be produced at exactly the same rates. This small production asymmetry it is estimated could be approximately 1%. It can also be measured from the data, as discussed in the LHCb paper.
- Detection asymmetry. The LHCb detector could be more efficient for detecting either the B^+ or the B^- final states. This is because the positive and negative kaons will be bent by the magnet indifferent directions in the detector. If the efficiency of the detector is higher in one region than another this will lead to higher efficiencies for K^+ or K^- and hence for B^+ or B^-. For this reason the magnetic field of the LHCb detector is regularly reversed. You used data in this analysis in which the magnetic field was both up and down and hence the effect will (partially) cancel. By comparing results for the two magnet polarities separately you can check the size of this effect. When loading the data above both polarities were combined, you can instead load them independently to measure the difference between the two datasets.
- Analysis technique. The analysis technique you have used may bias the result. A major simplification we made in the analysis above was to neglect 'background' events. We imposed a selection to select a sample of predominantly signal events but have not accounted for the effect of the residual background events.

Using mass sidebands

One source of 'background' events arises from random combinations of tracks in events that happen to fake the 'signal' characteristics. These events will not peak in the mass distribution at the mass of the B meson but rtaher will have a smoothly varying distribution. Looking at the number and distribution of of events away from the mass peak can allow you to estimate the number of background events under the mass peak.

Fitting distributions

The next level of sophistication in the analysis requires fitting the distributions of events that are observed in the B mass distribution in order to estimate the yield of signal events and background events. You can see how this is done in the LHCb paper on the analysis. Fitting can be performed using the CERN root framework.

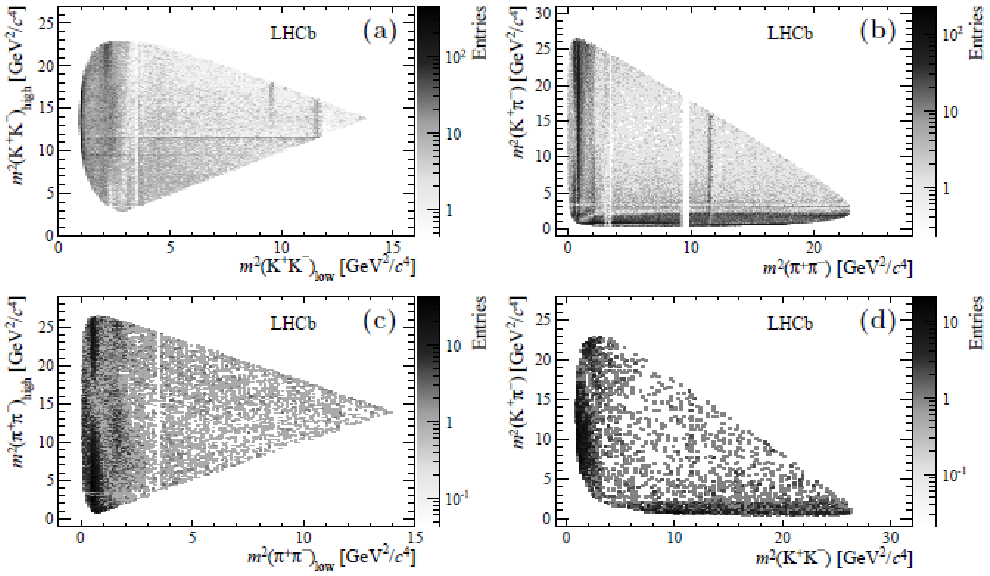


Figure 2: Dalitz plot distributions of (a) $B^{\pm} \to K^{\pm}K^{+}K^{-}$, (b) $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$, (c) $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and (d) $B^{\pm} \to \pi^{\pm}K^{+}K^{-}$ candidates. The visible gaps correspond to the exclusion of the J/ψ (in the $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ decay) and D^0 (all plots, except for the $B^{\pm} \to \pi^{\pm}K^{+}K^{-}$ decay) mesons from the samples.

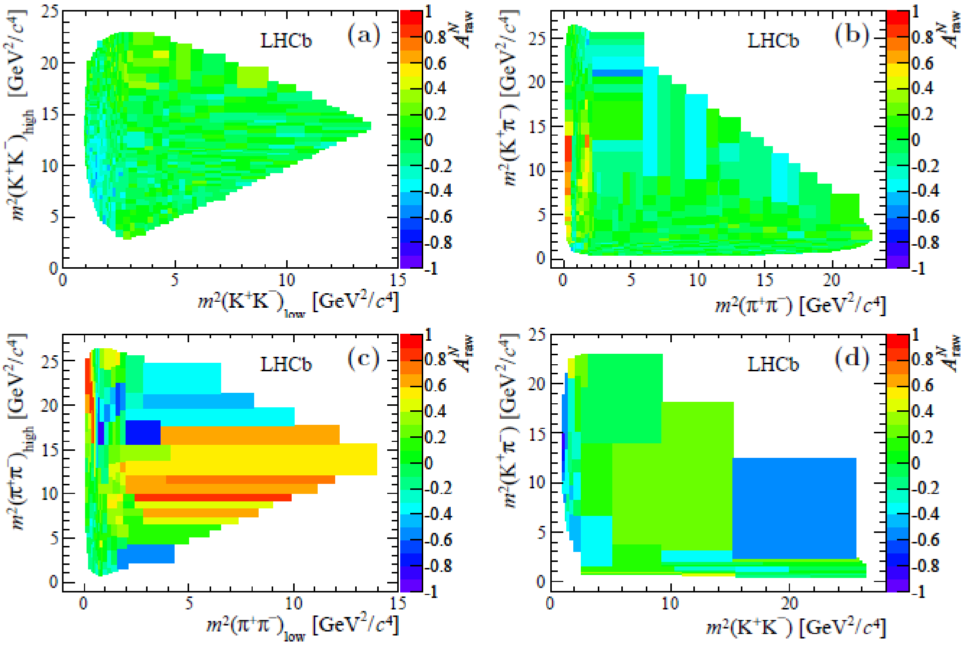


Figure 3: (colour online) Measured A^N_{raw} in Dalitz plot bins of background-subtracted and acceptance-corrected events for (a) $B^{\pm} \to K^{\pm}K^{+}K^{-}$, (b) $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$, (c) $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and (d) $B^{\pm} \to \pi^{\pm}K^{+}K^{-}$ decays.

R. Aaij *et al.* (LHCb Collaboration) Phys. Rev. D 90, 112004