| Statistics-CS | Random Variables, PDF and CDF |
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| Tutorial 2 |  |

1. Let $X$ denote the number of boys in a randomly selected three-child family. Assuming that boys and girls are equally likely, construct the probability distribution of $X$.
2. The probability distribution of the random variable $\mathrm{X}=$ number of changes in major is shown below:

| $X=\#$ changes in major | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Probability | 0.135 | 0.271 | 0.271 | 0.180 | 0.090 | 0.036 | 0.012 | 0.003 | 0.002 |

a) What is the probability that a college student will change majors at most once?
b) John's parents are concerned that he has decided to change his major for the second time and claims that it is not unusual. Find the probability that a randomly selected college student will change his major as often as or more often than John?
c) What is the probability that a student will change majors 5 or 6 times?
d) What is the probability that a student will change majors at least once?
3. You apply for a job in a factory that claims that the average salary is above $3,500 €$. With a sad surprise you see that your first earning is $2,000 €$. Do you have a strong proof that the ad was a fake? Construct a probability distribution for a random variable that describes salary for a staff that consist of 100 workers with $2,000 €, 10$ managers that earn $10,000 €$ and two directors with $50,000 €$.
4. The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between zero and 15 minutes, inclusive.
a) What is the probability that a person waits fewer than 12.5 minutes?
b) On the average, how long must a person wait? Find the mean, $\mu$, and the standard deviation, $\sigma$.
c) Ninety percent of the time, the time a person must wait falls below what value?
5. Given the Random Variable X has density function:

$$
f(x)=\left\{\begin{array}{cc}
2 x & 0<x<a \\
0 & \text { otherwise }
\end{array}\right.
$$

a) Determine a
b) Find $\mathrm{P}\left(\frac{1}{2}<\mathrm{X}<\frac{3}{4}\right)$ and $\mathrm{P}\left(-\frac{1}{2}<\mathrm{X}<\frac{1}{2}\right)$.
6. The length of time $X$, needed by students in a particular course to complete a 1 hour exam is a random variable with PDF given by

$$
f(x)=\left\{\begin{array}{c}
\left(k\left(x^{2}+x\right), \text { if } 0 \leqslant x \leqslant 1\right. \\
0, \quad \text { otherwise }
\end{array}\right.
$$

a) Find the value of $k$ that makes $f(x)$ a PDF?
b) Find the CDF.
c) Use CDF to find $P(X \leq 1)$ and $\mathrm{P}(X \leq 2)$
d) Find the probability that that a randomly selected student will finish the exam in less than half an hour.
e) Find the mean time needed to complete a 1 hour exam and standard deviation of X.

