Physics at LHCb

I. The LHCb experiment
II. Heavy flavour physics
III. Measurements @ LHCb
IV. Plans for Upgrade 1 and 2
V. How to do precise measurements:
   - mass and momentum resolution
   - proper time-life

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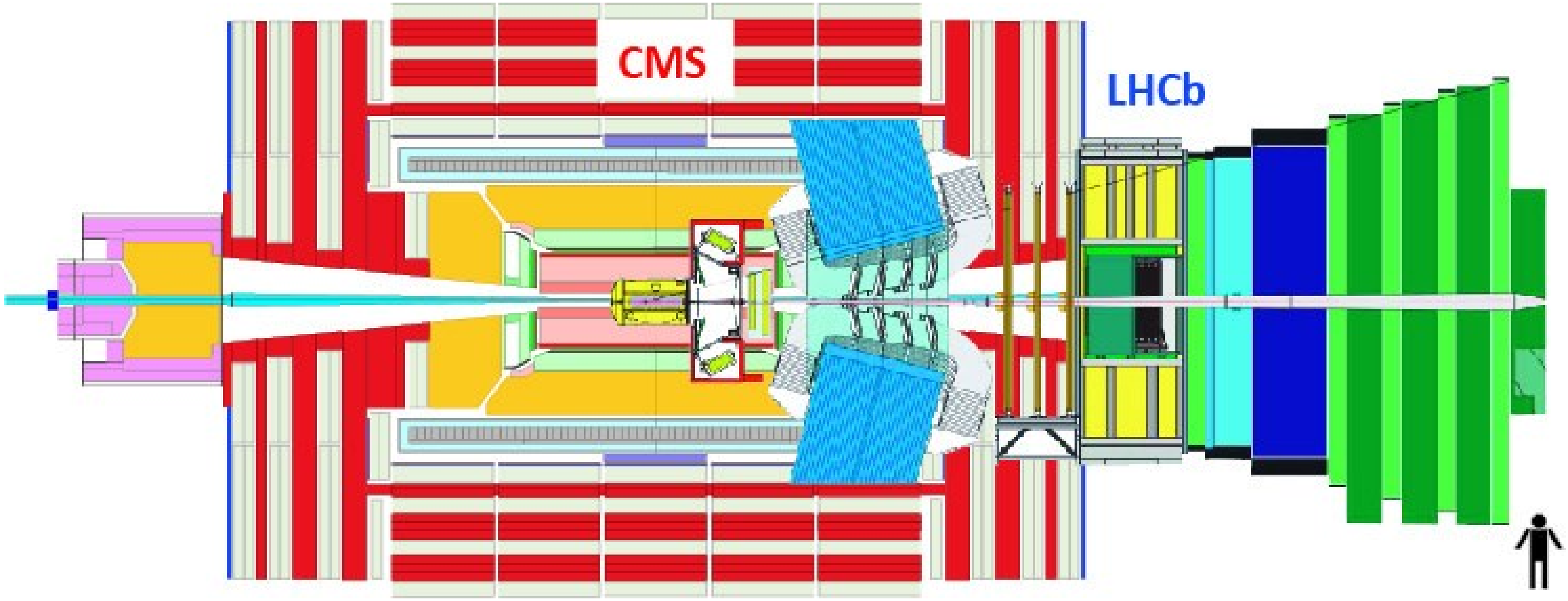
I. LHCb experiment

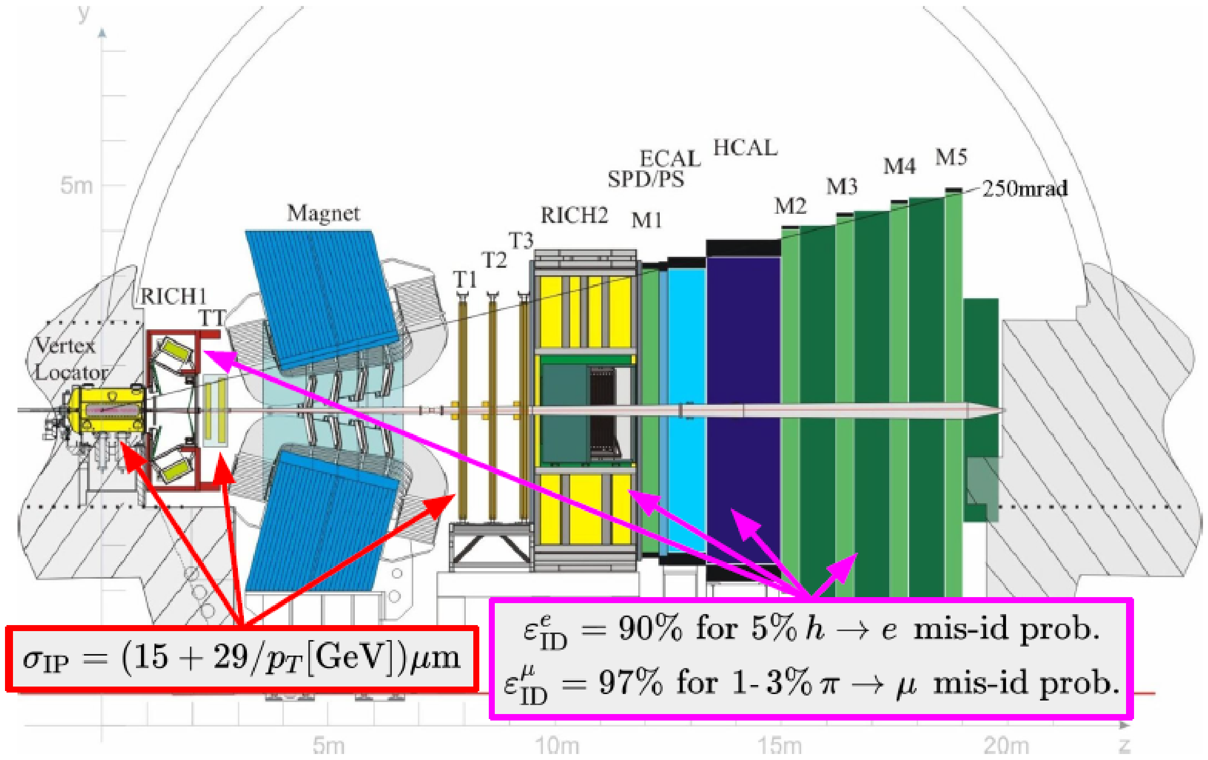
I. LHCb experiment for heavy flavour physics

LHCb – single arm spectrometer



Why spectrometer?

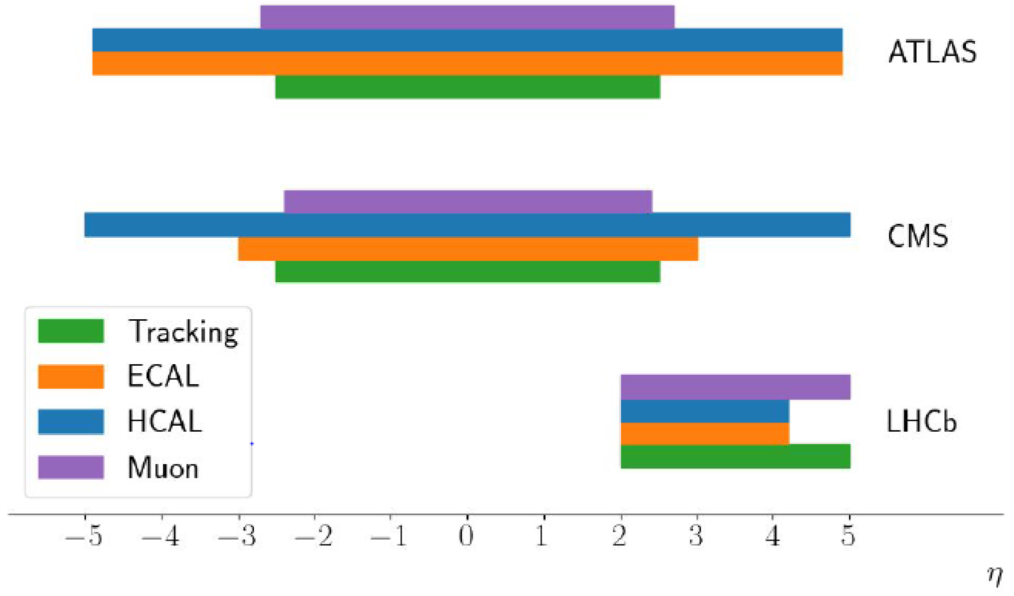
The experiment – LHCb spectrometer



$\sigma_{\mathrm{IP}} = (15 + 29/p_T[\mathrm{GeV}])\mu\mathrm{m}$

$\varepsilon^{e}_{\mathrm{ID}} = 90\%$ for 5% $h \to e$ mis-id prob.

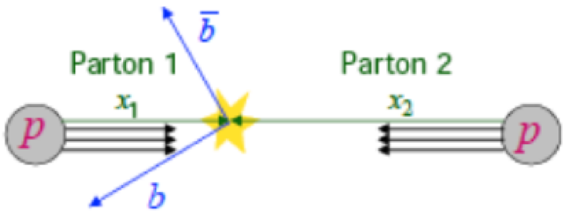
$\varepsilon^{\mu}_{\mathrm{ID}} = 97\%$ for 1-3% $\pi \to \mu$ mis-id prob.



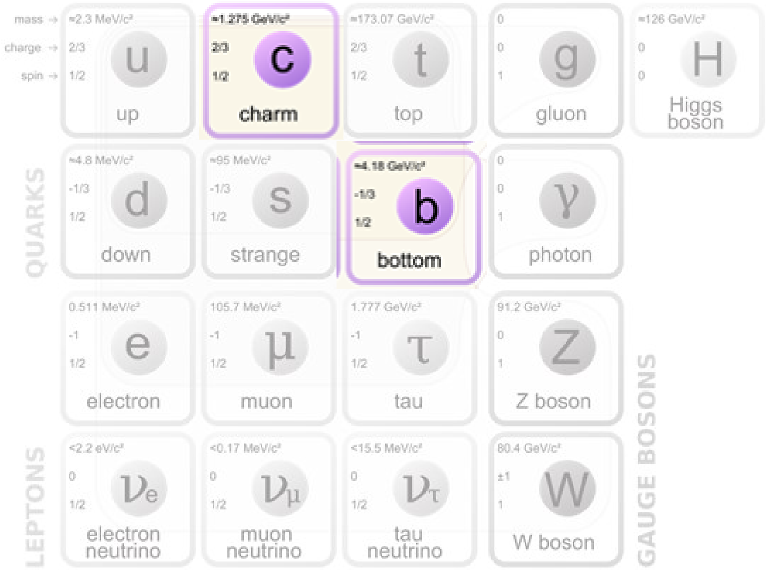
- Detector in the forward region with excellent momentum and vertex resolutions
- Coverage is complementary to ATLAS and CMS (with some overlapping at low pseudorapidity)

LHCb experiment for heavy flavour physics

LHCb experiment is dedicated for studying flavour physics at LHC.
Especially CP violation and rare decays of beauty and charm mesons.

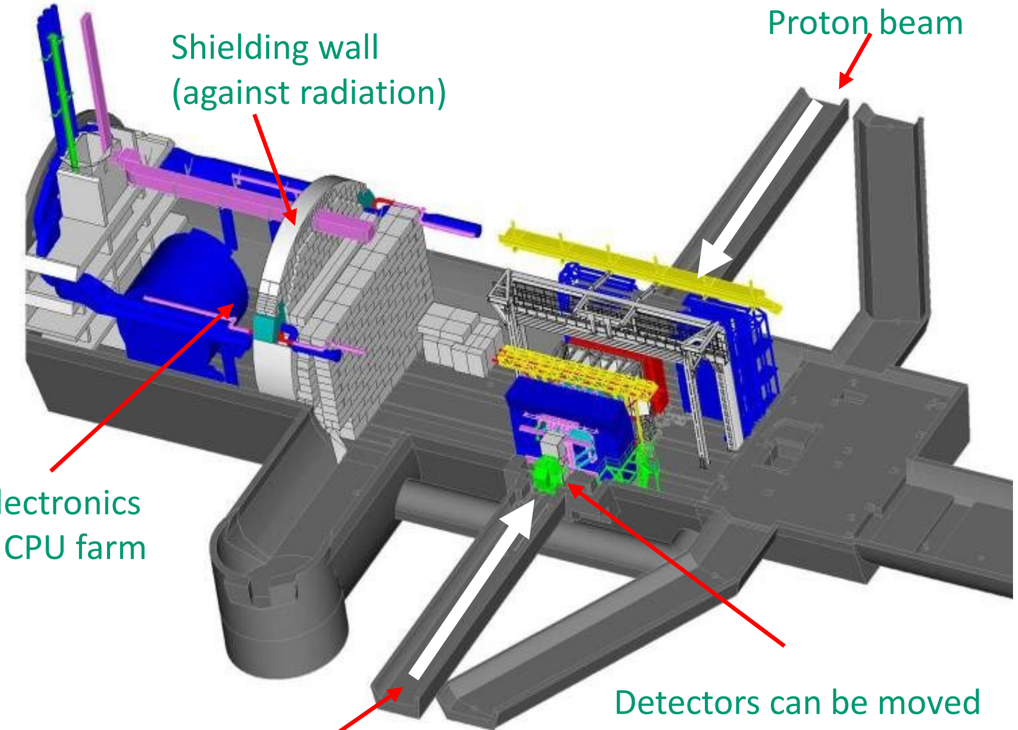


Standard Model



entrance

Cavern 100m below surface

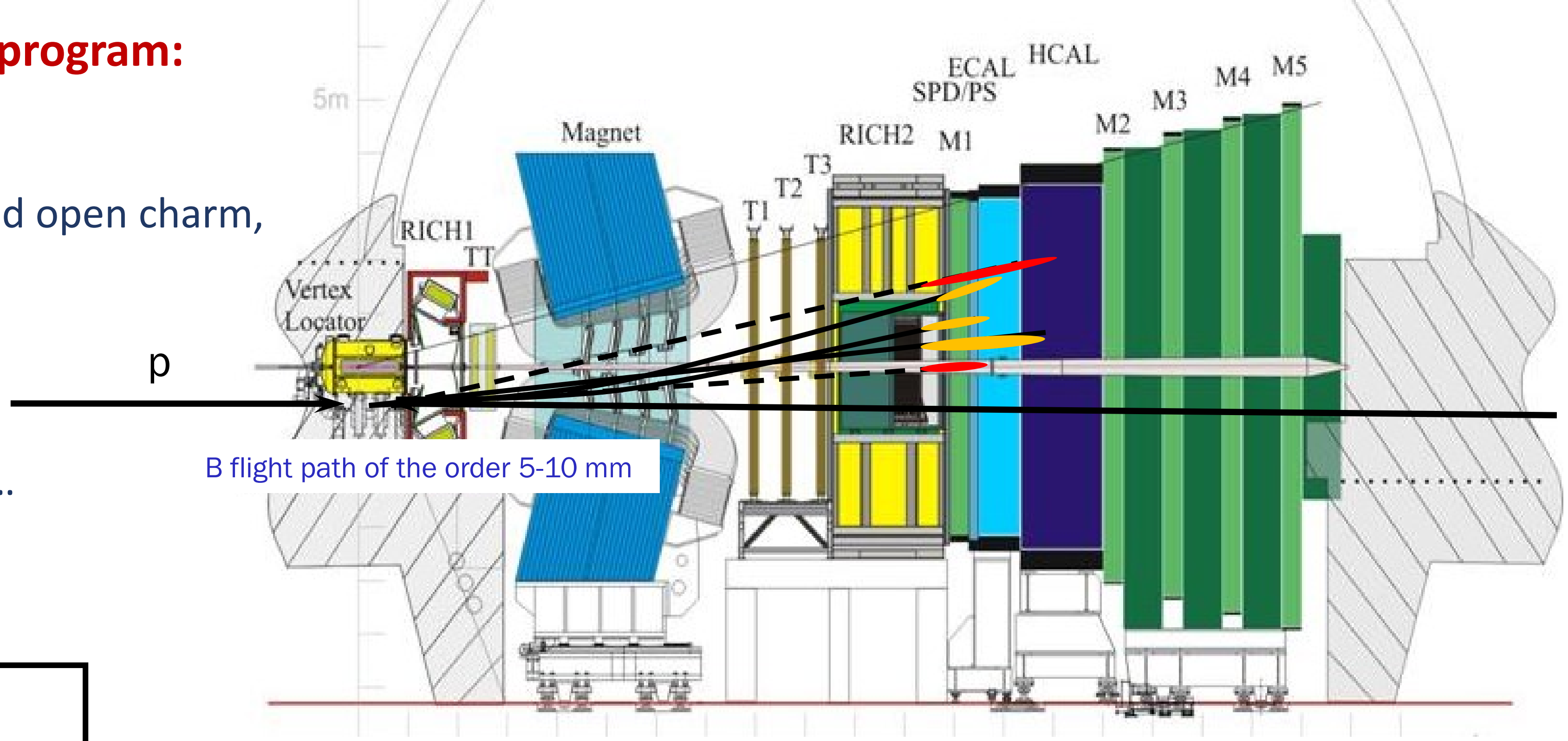


Physics program

The experiment – LHCb spectrometer

Physics program:

- CP Violation ,
- Rare B decays,
- B decays to charmonium and open charm,
- Charmless B decays,
- Semileptonic B decays,
- Charm physics
- B hadron and quarkonia
- QCD, electroweak, exotica …



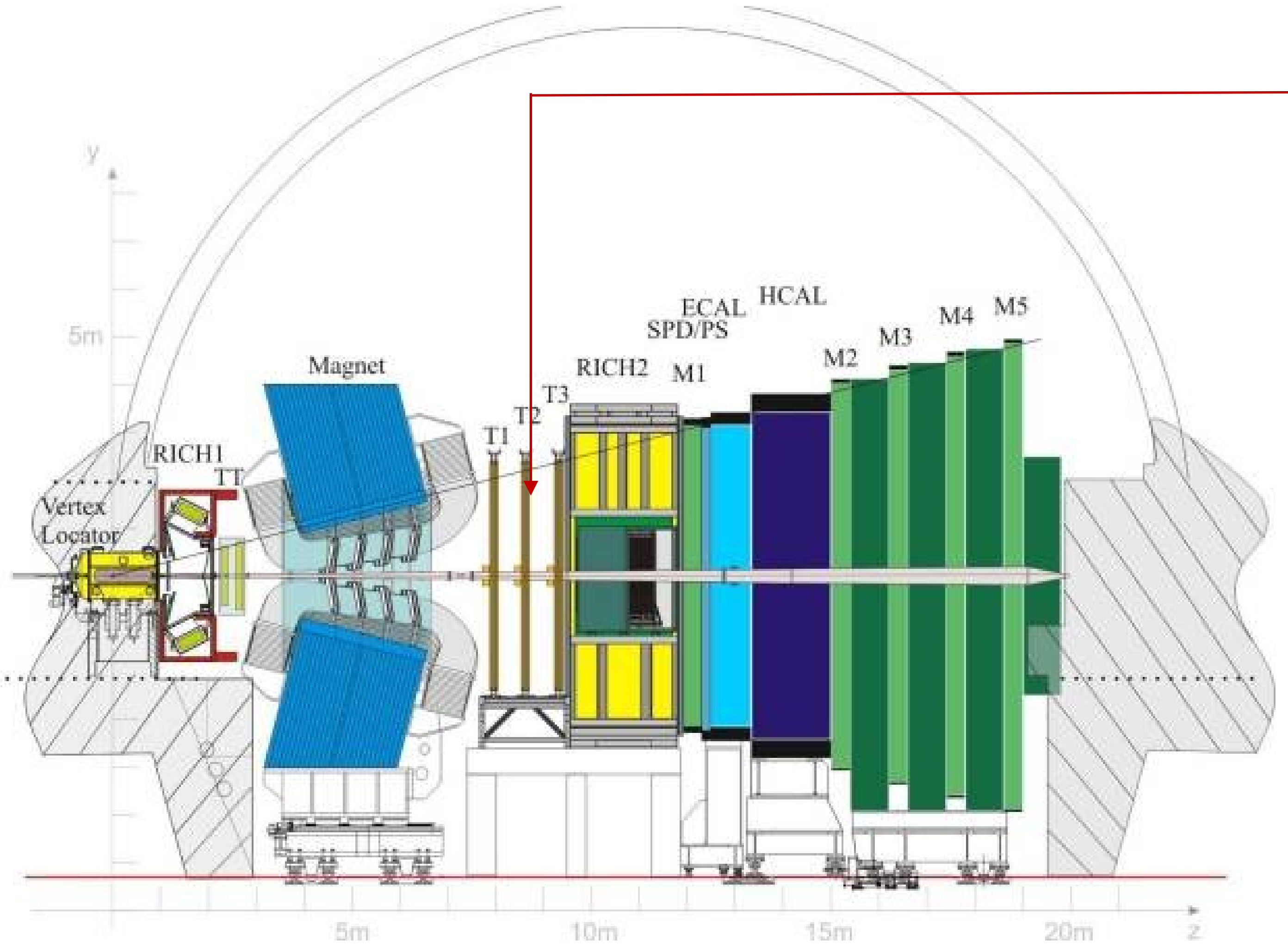
Tracking:
Silicon & Straw tubes
Magnetic field

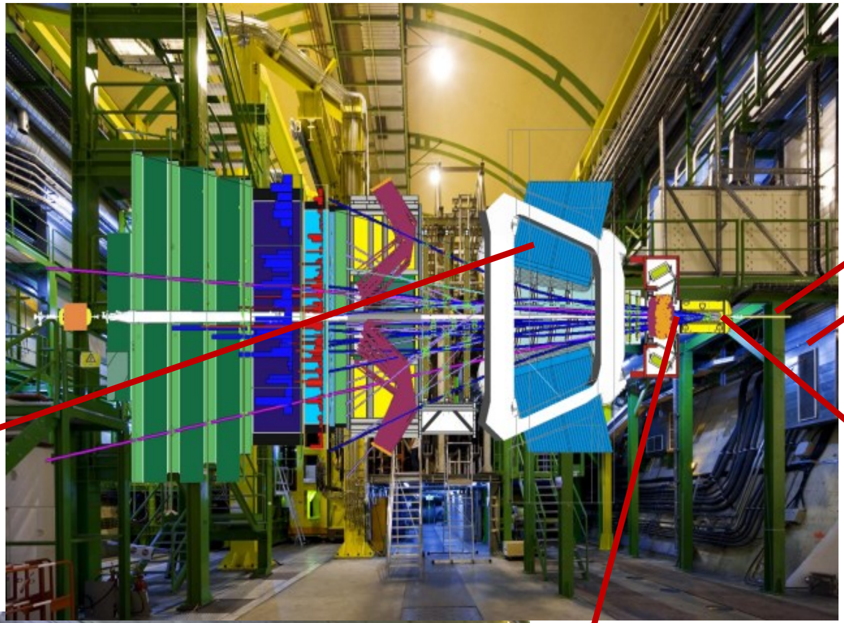
Vertexing:
High precision silicon detectors (10μm position resolution) very close to collision point

RICH performance:
Cherenkov radiation.
Measures velocity, combine with momentum to get mass
Particle identification in p range 1-100 GeV
π, K ID efficiency > 90%, misID<~10%

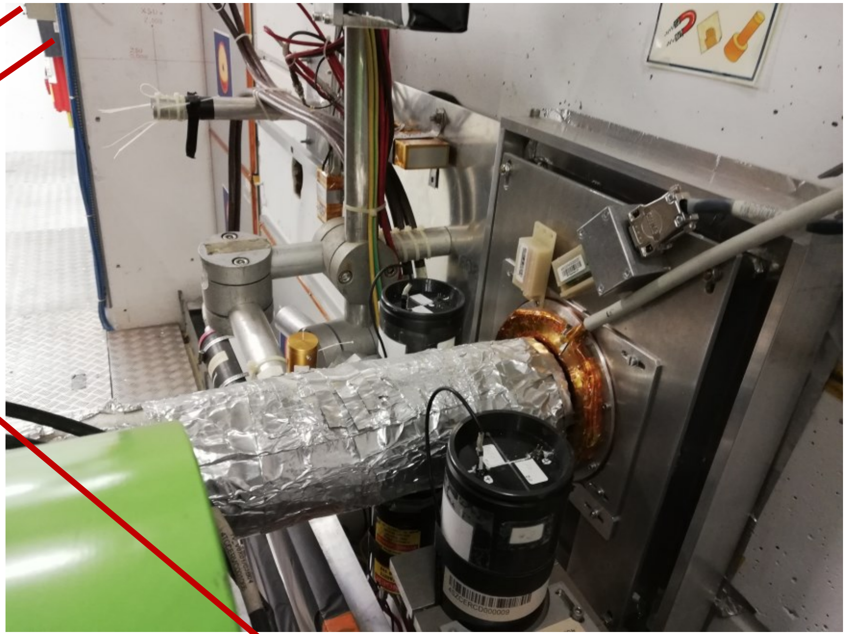
Calorimeters:
Electromagnetic &
Hadronic calorimeters
- Critical (with muons) for triggering

LHCb spectrometer

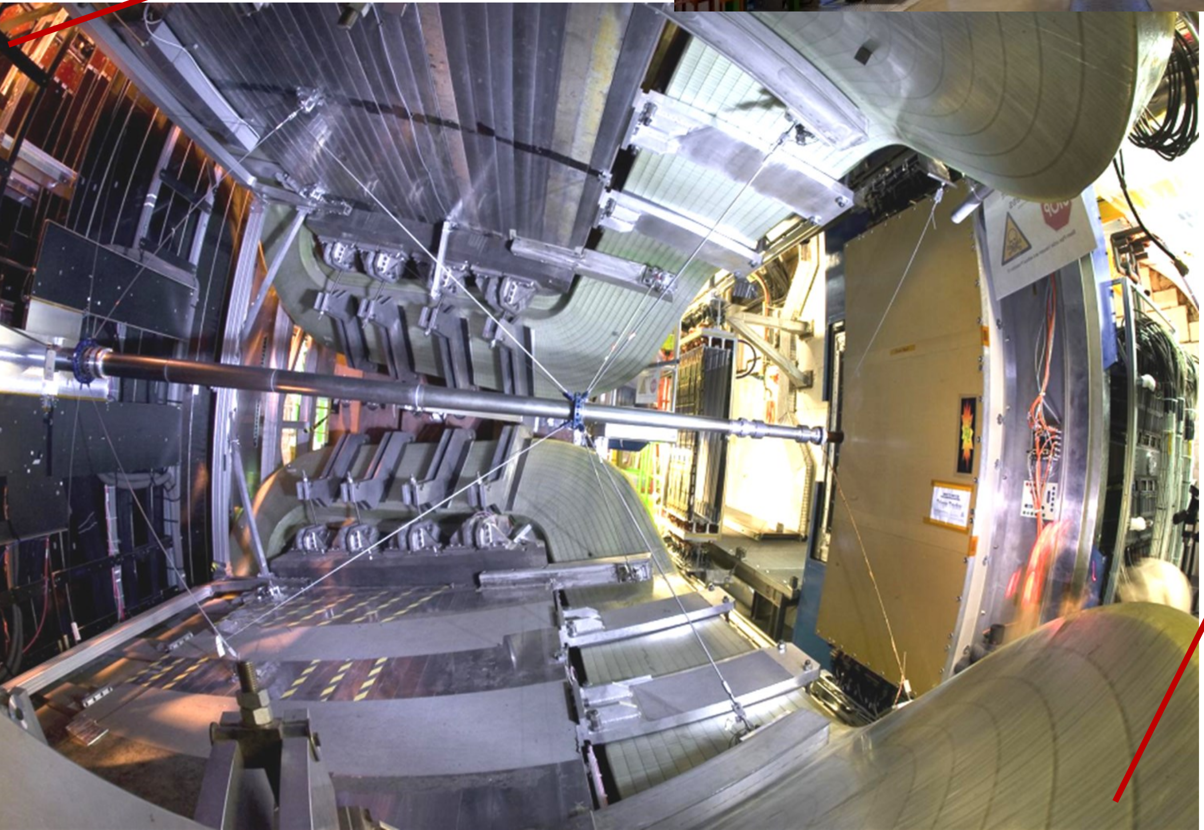




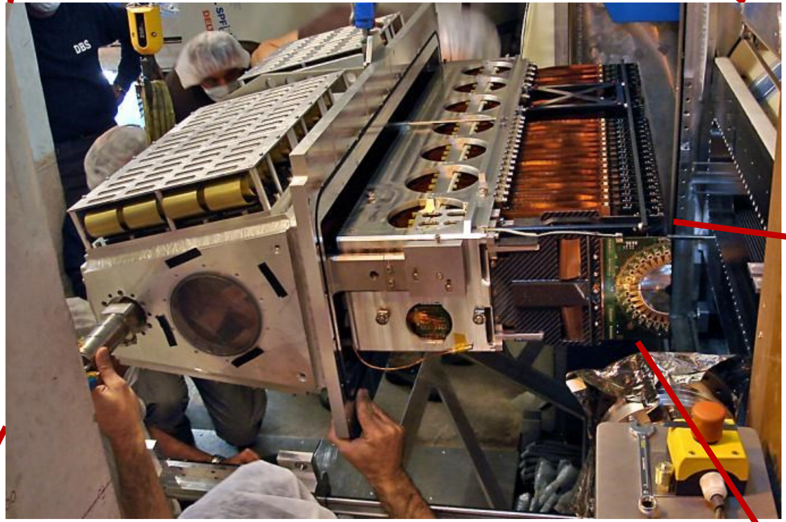
Beam pipe

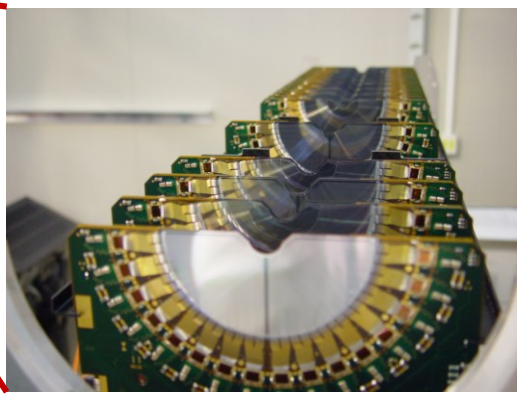


Inside the magnet



Vertex Locator



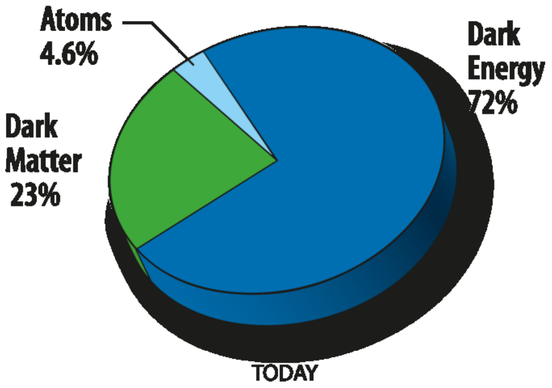


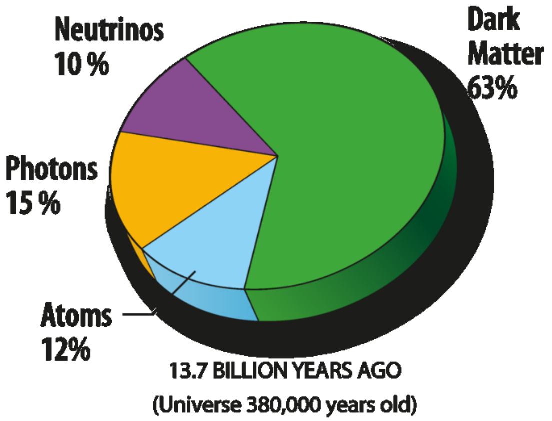
II. Heavy flavour physics

1. Discrete symetries C and P
2. Standard Model Flavours
3. Dirac equation
4. Three ways of CP Violation

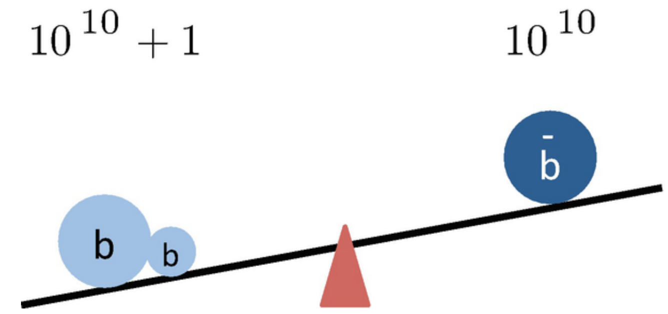
Puzzles in modern physics

What is the Universe made of?

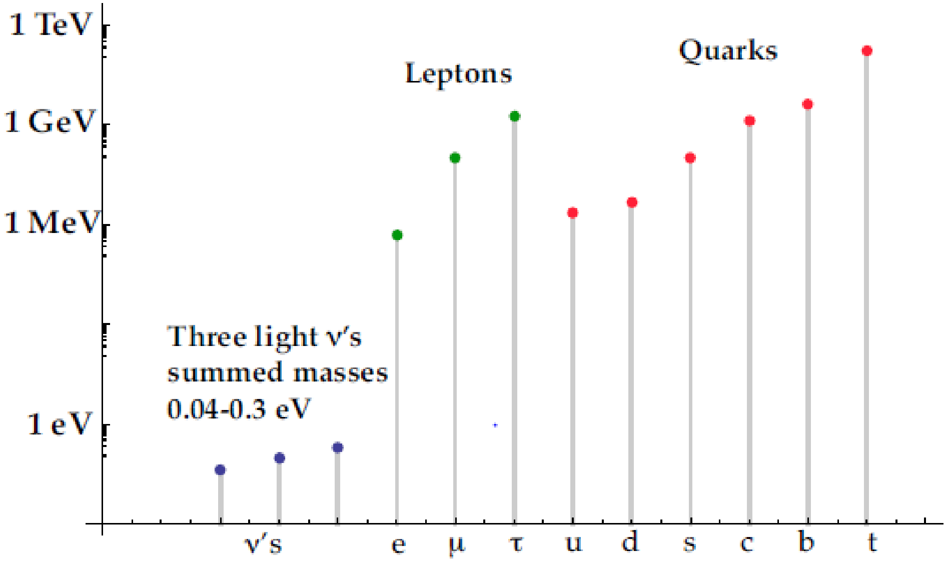




Baryon asymmetry



Mass hierarchies

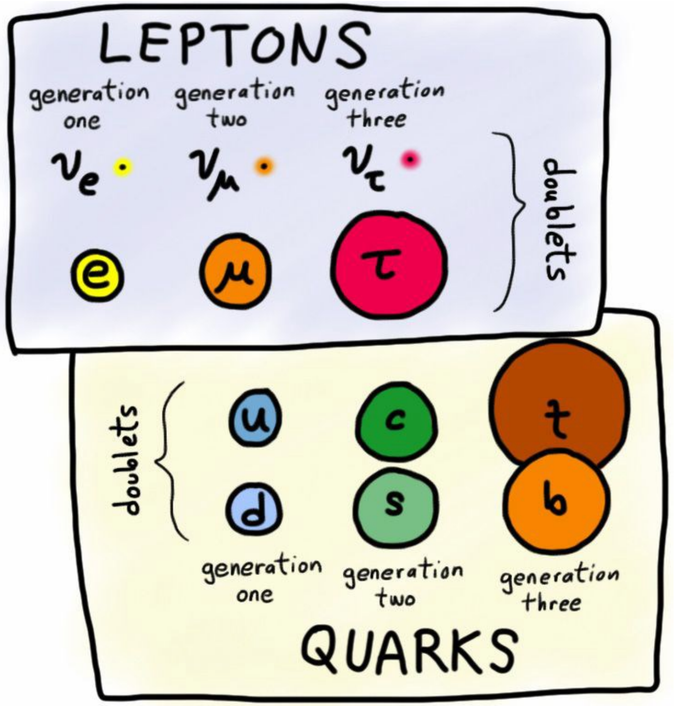


What is Flavour Physics?

CERNCOURIER

In 1971, at a Baskin-Robbins ice-cream store in Pasadena, California, Murray Gell-Mann and his student Harald Fritzsch came up with the term “flavour” to describe the different types of quarks. From the three types known at the time – up, down and strange – the list of quark flavours grew to six. A similar picture evolved for the leptons: the electron and the muon were joined by the unexpected discovery of the tau lepton at SLAC in 1975 and completed with the three corresponding neutrinos. These 12 elementary fermions are grouped into three generations of increasing mass. Camalich & Zupan 2019

Just as ice cream has both colour and flavour, so do quarks!



Flavour physics refers to the study of the interactions that distinguish between the fermion generations.

Matter-antimater difference

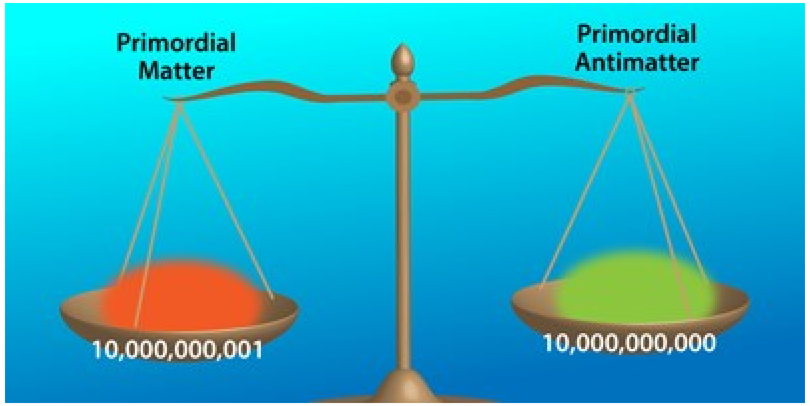
1. A long, long time ago, in the early Universe, there were an equal number of baryons and antibaryons…..

   High energy photons constantly produce protons and antiprotons which later annihilate:

$$\gamma + \gamma \rightleftharpoons p + \bar{p}$$

2. Then comes the time when temperature decreases and photons have not enough energy for particle creation.
3. As the Universe expanded the density of baryons and antibaryons decreased and annihilation was less and less probable.
4. The number of baryons and antibaryons was equal and related to number of photons:

$$n_B = n_{\bar{B}} \sim 10^{-18}\, n_\gamma$$



5. Meanwhile in the experiment…

… we observe that the Universe is dominated by baryons:

$$n_B - n_{\bar{B}} \sim 10^{-9}\, n_\gamma$$

It means that in order to generate this asymmetry we need to have $10^9 + 1$ baryons annihilating with 10^9 antibaryons (one baryon survives)

CPV might help?

Matter-antimater difference

Sakharov conditions for matter-antimatter asymmetry of the universe (1967):

1. **There must be a process that violates baryon number conservation.**

   Proton – the lightest baryon should decay. So far this is unobserved, the lifetimes of proton is greater than 10^{35} years.

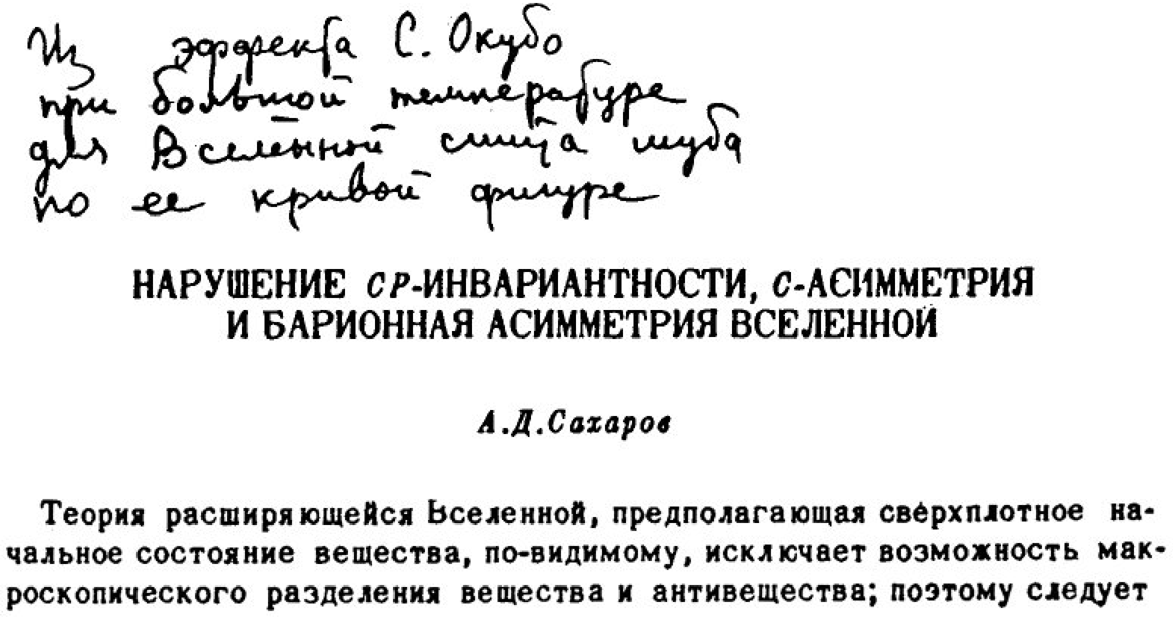
2. **Both C and CP symmetries should be violated.**

   $$p \neq \overline{p}$$

   This the subject of the following story.

3. **These two conditions must occur in a phase when there was no thermal equilibrium.**

Otherwise $N_{baryons} = N_{\overline{baryons}}$



Из эффекта С. Окубо
при большой температуре
для Вселенной сшита шуба
по ее кривой фигуре

НАРУШЕНИЕ *CP*-ИНВАРИАНТНОСТИ, *C*-АСИММЕТРИЯ И БАРИОННАЯ АСИММЕТРИЯ ВСЕЛЕННОЙ

А.Д.Сахаров

Теория расширяющейся Вселенной, предполагающая сверхплотное начальное состояние вещества, по-видимому, исключает возможность макроскопического разделения вещества и антивещества; поэтому следует

Andrei Sakharov:

- „father” of Soviet hydrogen bomb
- Dissident
- Nobel Peace Prize Winner

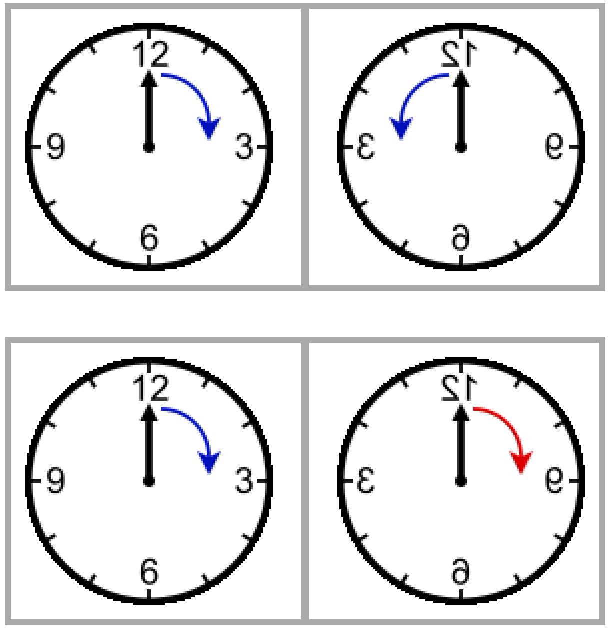
Parity Operator P

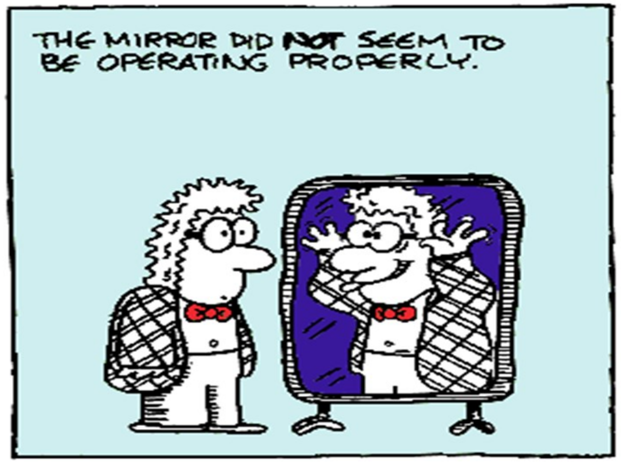
- The parity operator is a unitary operator that reverses the sign of spatial coordinates:

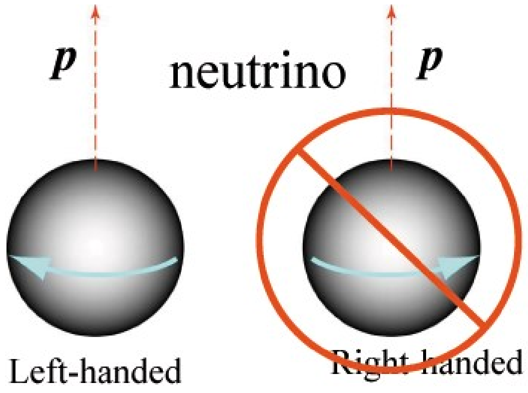
$$\hat{P}\,\Psi(\vec{r}) = \Psi(-\vec{r})$$

- In daily life we see no difference between „our world” and world in the mirror.
- In the world of particles – the difference is HUUGE!

Which clock violates P symmetry?



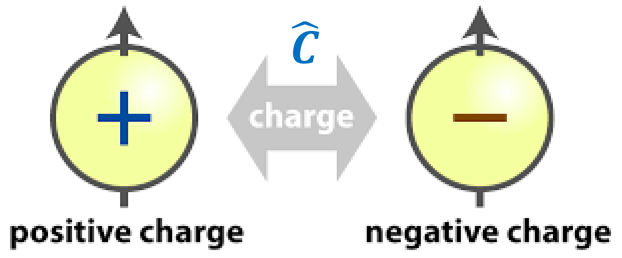




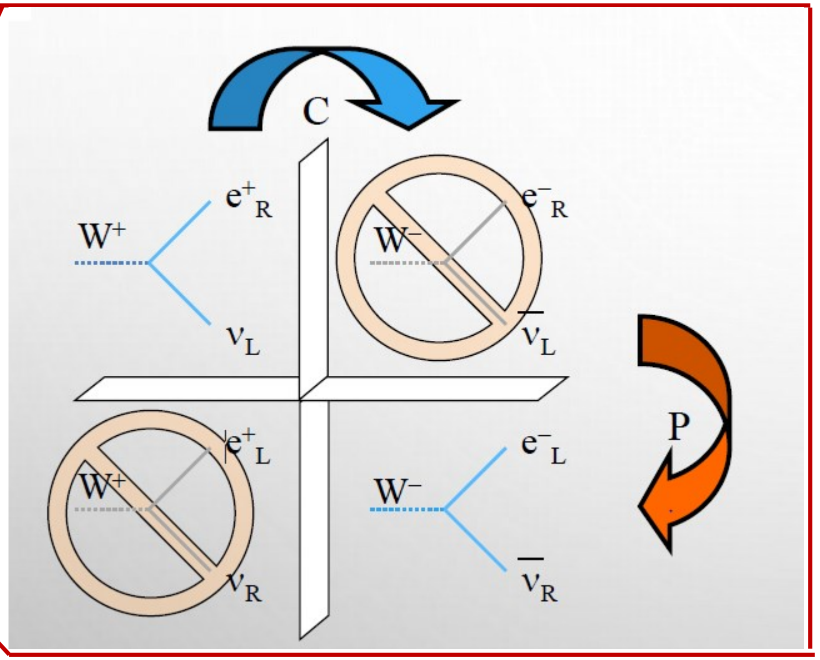
Charge C and combined CP Symmetry

- Charge conjugation $\hat{C}$ is a unitary operator that changes particle to antiparticle:

$$\hat{C}|\pi^0\rangle = +|\pi^0\rangle$$
$$\hat{C}|\gamma\rangle = -|\gamma\rangle$$
$$\hat{C}|e^-\rangle = |e^+\rangle$$



- Before weak interaction were studied, one cannot say in which side of the mirror lived or whether he/she was build of matter or antimatter.
- We call this „Strong and electromagnetic interactions conserve C and P symmetry, but weak interaction does not".
- Let's combine C and P together and see if neutrino is OK now:



Dirac equation

- Dirac (1928) formulated the alternative wave equation for relativistic particles as simple „square root" of the Klein-Gordon (K-G) equation:

$$\left(i\gamma^0\frac{\partial}{\partial t}+i\vec{\gamma}\cdot\nabla-m\right)\psi=0$$

$$-\frac{\partial^2}{\partial t^2}\Psi+\nabla^2\Psi=m^2\Psi$$

or in shorter notation:

$$\boldsymbol{(i\gamma^\mu\partial_\mu-m)\psi=0}$$

Dirac Equation in the covariant form

where: $\gamma^\mu=(\gamma^0,\gamma^1,\gamma^2,\gamma^3)$ are unknown coefficients to be defined….

$$\partial_\mu\equiv\frac{\partial}{\partial x^\mu}=\left(\frac{\partial}{\partial t},\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)$$

- To find how the $\boldsymbol{\gamma^\mu}$ should look like, first multiply the Dirac equation by its conjugate equation:

$$\psi^\dagger\left(-i\gamma^0\frac{\partial}{\partial t}-i\vec{\gamma}\cdot\nabla-m\right)\left(i\gamma^0\frac{\partial}{\partial t}+i\vec{\gamma}\cdot\nabla-m\right)\psi=0$$

what should restore the K-G equation.

- This leads to the conditions on the $\boldsymbol{\gamma^\mu}$:

$(\gamma^\mu)^2=1\ ,\ \mu,\nu=0,1,2,3$

$(\gamma^i)^2=-1\ ,\ i=1,2,3$

$\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=0$ for $\mu\neq\nu$,

or in terms of anticommutation relation: $\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}$

Four orthogonal solutions to the free particle Dirac equation:

$$\psi_i=u_i(E,\vec{p})e^{-i(Et-\vec{p}\cdot\vec{x})}$$

Dirac equation – solutions for moving particles

… where:

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix} \qquad u_2 = \begin{pmatrix} 0 \\ 1 \\ \frac{p_x - ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix} \qquad u_3 = \begin{pmatrix} \frac{p_z}{E-m} \\ \frac{p_x + ip_y}{E-m} \\ 1 \\ 0 \end{pmatrix} \qquad u_4 = \begin{pmatrix} \frac{p_x - ip_y}{E-m} \\ \frac{-p_z}{E-m} \\ 0 \\ 1 \end{pmatrix}$$

electron with energy

$$E = +\sqrt{m^2 + p^2}$$

$$\psi = u_{1,2}(p^\mu)\, e^{-i(Et - \vec{p}\cdot\vec{x})}$$

positron with energy

$$E = -\sqrt{m^2 + p^2}$$

$$\psi = u_{3,4}(p^\mu)\, e^{-i(Et - \vec{p}\cdot\vec{x})} \qquad p^\mu = (E, \vec{p})$$

Now we can take F-S interpretation of antiparticles as particles with positive energy (propagating backwards in time), and change the negative energy solutions $u_{3,4}$ to represent positive antiparticle (positron) spinors $v_{1,2}$:

$$v_1(E,\vec{p})\, e^{-i(Et - \vec{p}\cdot\vec{x})} \equiv u_4(-E, -\vec{p}) e^{-i(-Et + \vec{p}\cdot\vec{x})} = u_4(-E, -\vec{p}) e^{i(Et - \vec{p}\cdot\vec{x})}$$

$$v_2(E,\vec{p})\, e^{-i(Et - \vec{p}\cdot\vec{x})} \equiv u_3(-E, -\vec{p}) e^{-i(-Et + \vec{p}\cdot\vec{x})} = u_3(-E, -\vec{p}) e^{i(Et - \vec{p}\cdot\vec{x})}$$

reversing the sign of *E* and *p*

$$E = +\sqrt{m^2 + p^2}$$

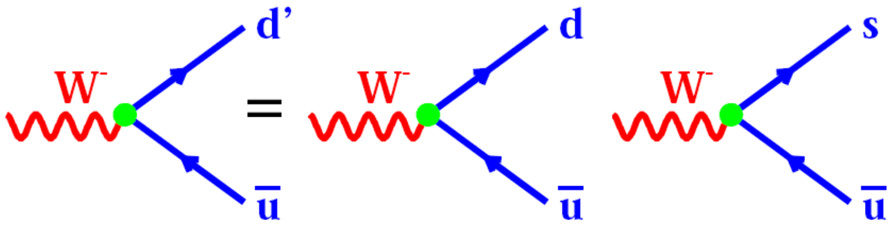
The u and v are solutions of:

$$\boldsymbol{(i\gamma^\mu p_\mu - m)u = 0} \quad \text{and} \quad \boldsymbol{(i\gamma^\mu p_\mu + m)v = 0}$$

Heavy flavour physics – 2 generation

Stany, które biorą udział w słabych oddziaływaniach są ortogonalnymi kombinacjami stanów o określonym zapachu, czyli:

oddz. słabe „widzą” zamiast kwarka d – jego stan będący kombinacją d i s:

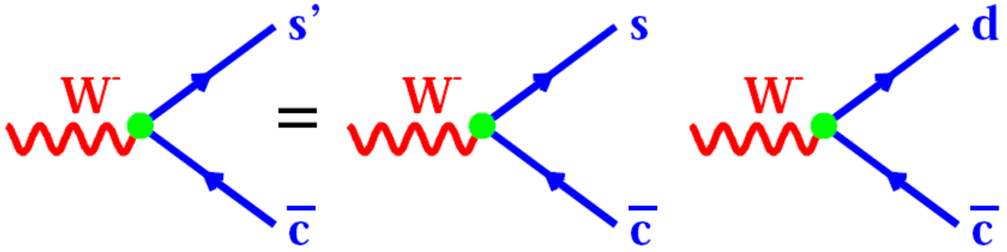


$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$d' = d\cos\theta_c + s\sin\theta_c$$
$$s' = s\cos\theta_c - d\sin\theta_c$$

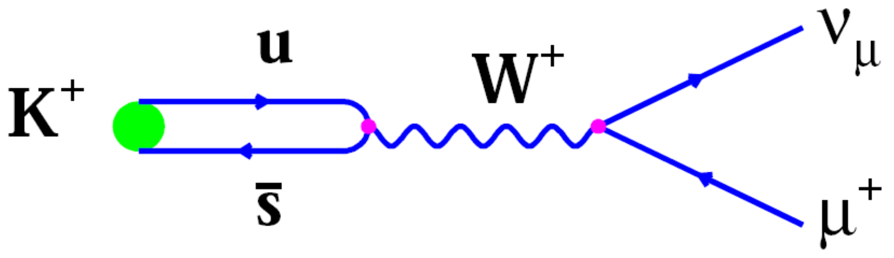
$$\theta_c = 13^\circ$$

W oddziaływaniach słabych częściej występują człony z $\cos\theta_c$, człony proporcjonalne do $\sin\theta_c$ są tłumione.



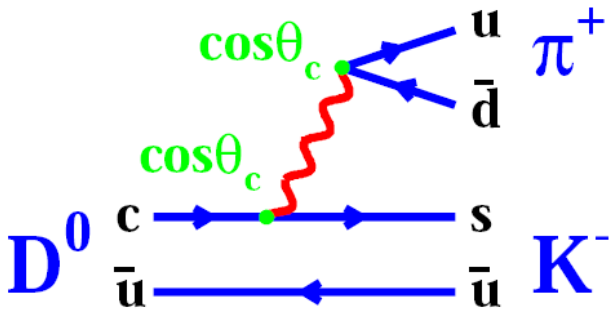
Heavy flavour physics – 2 generation

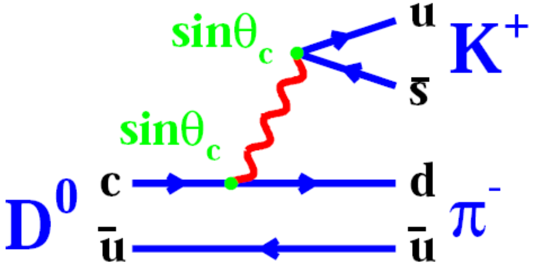
Kaon decay – change of generation – change of flavour!



M. Thomson

Decay of D mesons:

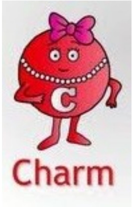
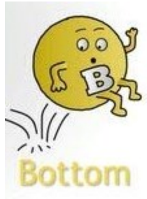


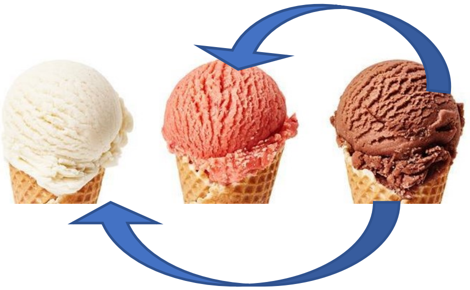


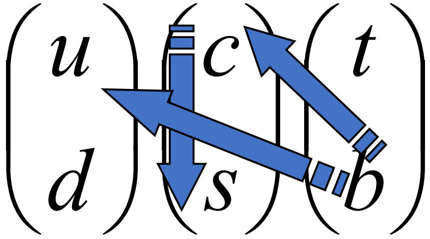
$$\frac{\Gamma(D^0 \to K^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)} = \frac{\sin^4\theta_c}{\cos^4\theta_c} \approx 0.0028$$

Heavy flavour physics – 3 generations

Heavy flavour physics deals with change of quarks' flavour.

Especially heavy quarks:  and  because they are heavy!

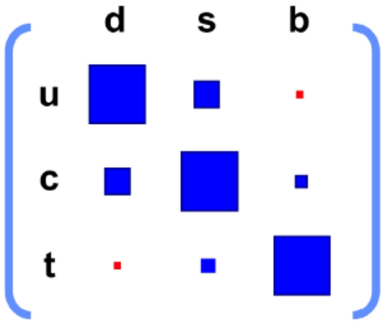




WEAK interactions!

Transitions between quarks are described by a (famous) CKM matrix:

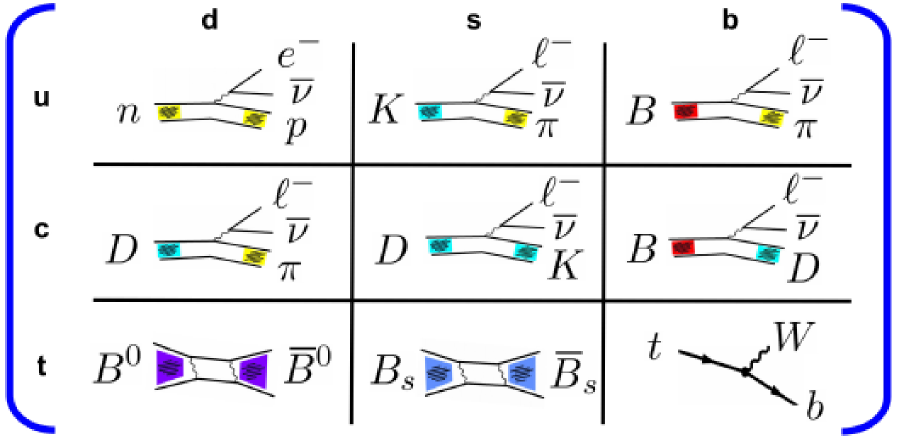
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{V_{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



V_{CKM} elements:

- are described within the Standard Model,
- are obtained experimentally !

V =



The CKM Matrix

1. In two generation system (1964) – one angle – no CP violation
2. Third generation proposed by Kobayashi & Maskawa (1973) opened the Pandora's box of new ideas how to measure CPV.
3. Many possible parametrizations:
   a) original K-M: $s_i = \sin\vartheta_i$; $s_i = \cos\vartheta_i$
   b) Standard representation (PDG proposal):

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

$$\begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{23} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \end{pmatrix}$$

The CKM matrix is described by three rotation angles and a complex phase.

CKM matrix parametrization

$$\begin{bmatrix}|d'\rangle\\|s'\rangle\\|b'\rangle\end{bmatrix} = \begin{bmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix}\begin{bmatrix}|d\rangle\\|s\rangle\\|b\rangle\end{bmatrix}$$

4. Wolfenstein parametrization (1983):
   a) matrix elements are expanded in terms of $\sin\vartheta_i \equiv \lambda$;
   b) from kaons sector we have $V_{us} = \lambda = 0.22$, and $V_{ud} = (1 - \lambda^2/2)$
   c) from B-lifetime: $V_{cb} = 0.04 - 0.006 = A\lambda^2$
   d) let's keep V_{ud}, V_{us}, V_{tb} real – expressed in term of four real parameters : λ, A, ρ, η
   e) the only complex components are in V_{ub} and V_{td} , third row/column are of order smaller (CPV effects) $A\lambda^3(\varrho - i\eta)$
   f) For the CP violation – one phase should be measurable, so η cannot be zero

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta + i\eta\frac{\lambda^2}{2}) \\ -\lambda & 1-\frac{\lambda^2}{2} - i\eta A\lambda^4 & A\lambda^2(1+i\eta\lambda^2) \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$|V_{td}|e^{-i\beta}$

$|V_{ub}|e^{-i\gamma}$

Higher order in CKM

5. Higher order in Wolfenstein parametrization:

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

$$\begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda-A^2\lambda^5(\rho+i\eta-\frac{1}{2}) & 1-\frac{\lambda^2}{2}-(\frac{1}{8}+\frac{A}{2})\lambda^4 & A\lambda^2 \\ A\lambda^3[1-(\rho+i\eta)(1-\frac{\lambda^2}{2})] & -A\lambda^2-A\lambda^4(\rho+i\eta-\frac{1}{2}) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

$|V_{ts}|e^{-i\beta_s}$

$\boldsymbol{\beta}$ and $\boldsymbol{\beta_s}$ are weak phases in B^0 and B_S^0 mixing,

$\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are the CKM angles (see next slides).

They are ones of the most important observables in experimental heavy flavour physics

Unitarity of CKM matrix

The CKM matrix is unitary $V_{CKM}{}^{-1} = V_{CKM}{}^{\dagger}$ – this give us 12 orthogonality conditions:

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \qquad V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \qquad V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

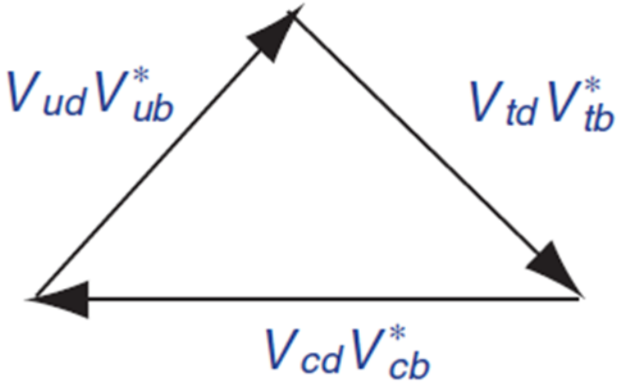
$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1 \qquad V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} = 0$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \qquad V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 \qquad V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \qquad V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0$$

The orthogonality conditions can be regarded as a triangle condition – CKM matrix elements are complex numbers, so their sum is simply a sum of three vectors:



Unitarity of CKM matrix

But most of them have magnitudes of very different size and are currently useless from experimental point of view :

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$V^*_{ud}V_{cd} + V^*_{us}V_{cs} + V^*_{ub}V_{cb} = 0 \qquad \lambda,\ \lambda,\ \lambda^5$$

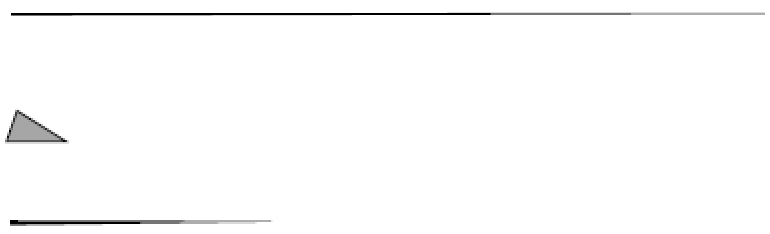
$$V^*_{ud}V_{td} + V^*_{us}V_{ts} + V^*_{ub}V_{tb} = 0 \qquad \lambda^3,\ \lambda^3,\ \lambda^3$$

$$V^*_{cd}V_{td} + V^*_{cs}V_{ts} + V^*_{cb}V_{tb} = 0 \qquad \lambda^4,\ \lambda^2,\ \lambda^2$$

$$V_{ud}V^*_{us} + V_{cd}V^*_{cs} + V_{td}V^*_{ts} = 0 \qquad \lambda,\ \lambda,\ \lambda^5$$

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0 \qquad \lambda^3,\ \lambda^3,\ \lambda^3$$

$$V_{us}V^*_{ub} + V_{cs}V^*_{cb} + V_{ts}V^*_{tb} = 0 \qquad \lambda^4,\ \lambda^2,\ \lambda^2$$



The most attractive are two triangles:

$$\boxed{V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0} \qquad V^*_{ud}V_{td} + V^*_{us}V_{ts} + V^*_{ub}V_{tb} = 0$$

Unitarity of CKM matrix

But still two promising left:

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$V^*_{ud}V_{td} + V^*_{us}V_{ts} + V^*_{ub}V_{tb} = 0 \qquad \lambda^3, \lambda^3, \lambda^3$$

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0 \qquad \lambda^3, \lambda^3, \lambda^3$$

„The" unitary triangle!

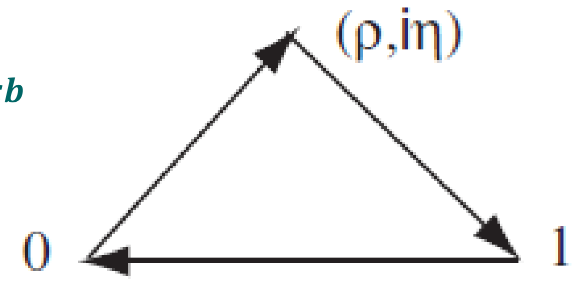
Using Wolfenstein parametrization, we can draw them on complex plane :

$$V_{ud}V^*_{ub} = A\lambda^3(1-\lambda^2/2)(\varrho + i\eta)$$

$$V_{cd}V^*_{cb} = -A\lambda^3$$

$$V_{td}V^*_{tb} = A\lambda^3(1-\varrho-i\eta)$$

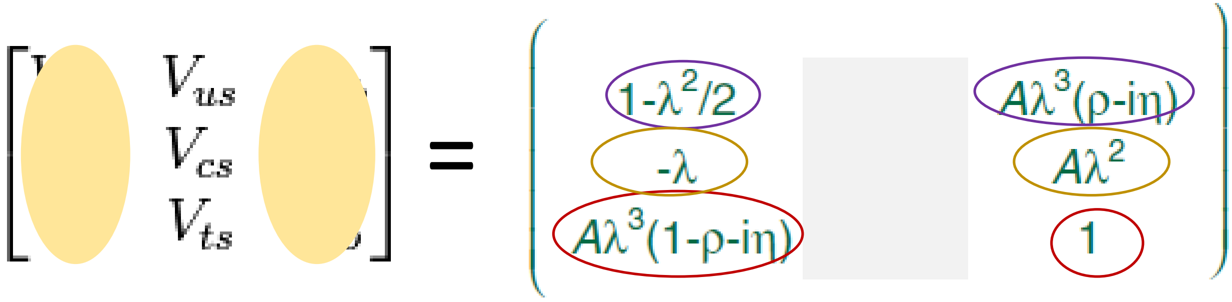
if sides are divided by $V_{cd}V^*_{cb}$ the UT looks like that:



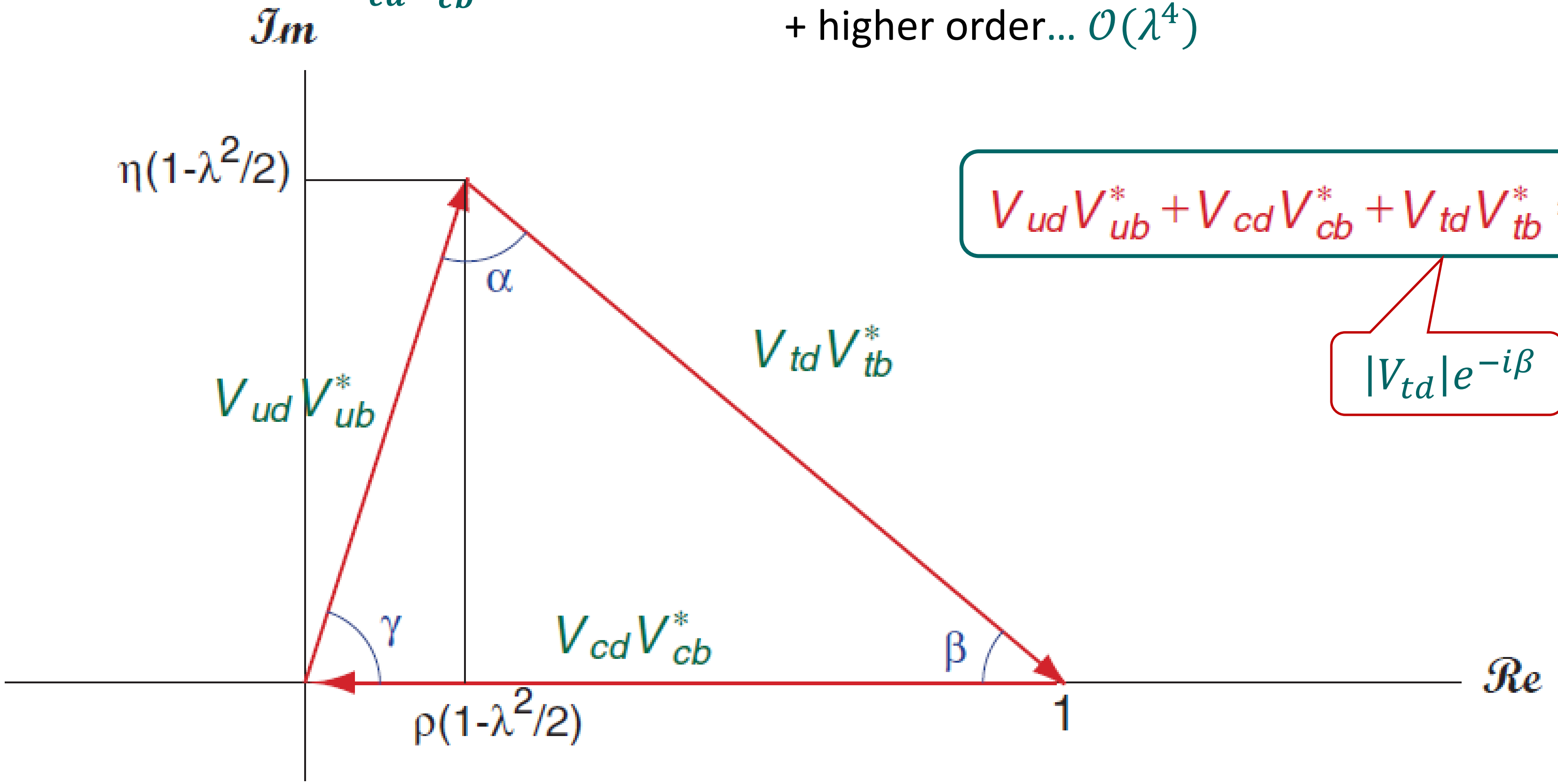
The unitary triangle

Try your vector algebra…

$$V_{ud}V_{ub}^* = A\lambda^3(1-\lambda^2/2)(\varrho + i\eta)$$
$$V_{td}V_{tb}^* = A\lambda^3(1-\varrho - i\eta)$$
$$V_{cd}V_{cb}^* = -A\lambda^3$$



+ higher order… $\mathcal{O}(\lambda^4)$



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$|V_{td}|e^{-i\beta}$

$$\alpha = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

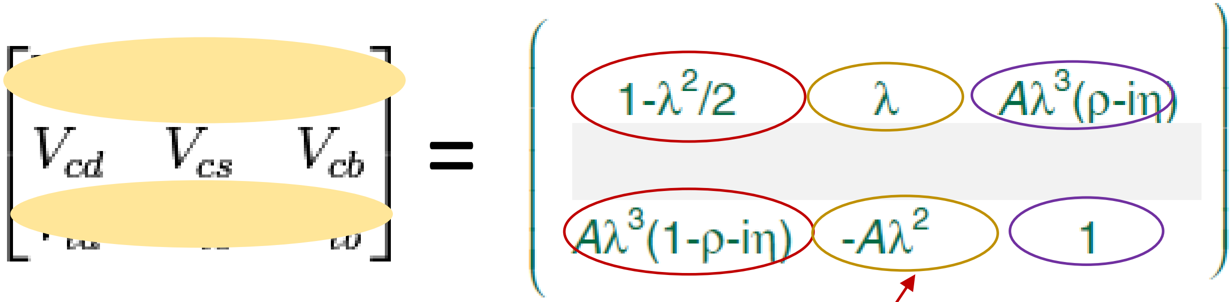
$$\beta = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

And another unitarity triangle

and more complex example…

$$V_{ub}V_{tb}^{*} = A\lambda^{3}(\varrho + i\eta)$$
$$V_{ud}V_{td}^{*} = A\lambda^{3}\left(1-\lambda^{2}/2\right)(1-\varrho-i\eta)$$
$$V_{us}V_{ts}^{*} = -A\lambda^{3}$$



+ higher order… $\mathcal{O}(\lambda^4)$

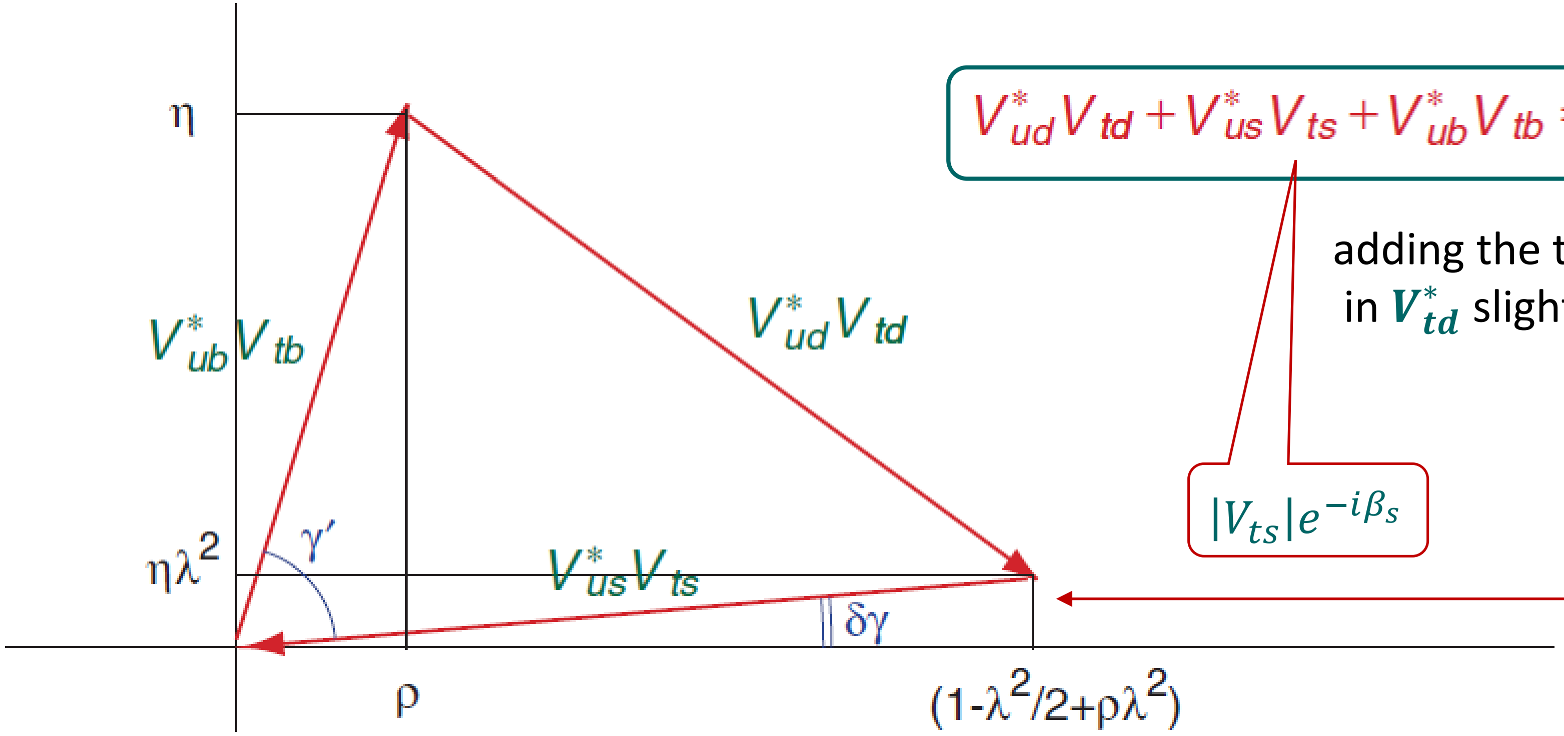
$-iA\lambda^4\eta$

$$V_{ud}^{*}V_{td} + V_{us}^{*}V_{ts} + V_{ub}^{*}V_{tb} = 0$$

$|V_{ts}|e^{-i\beta_s}$

adding the term $\mathcal{O}(\lambda^4)$ in V_{td}^{*} slightly tilts the Δ

precise measurements can prove this!



Time evolution of neutral mesons (K^0 story)

1. Meson K^0 can decay into all, allowed by energy-momentum conservation, states.
2. The exponential decay law leads to the time dependence of the wave function:

$$|K^0(t)\rangle = |K^0\rangle\, e^{-\frac{\Gamma t}{2}}\, e^{-imt}$$

time evolution of a stable state with mass m, $m = E$

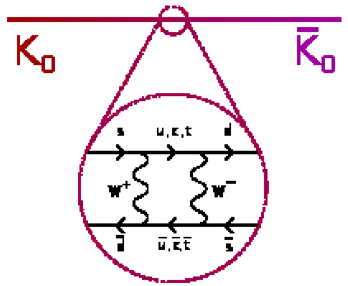
total width such that probability of finding an undecayed meson at time t is:

$$|\langle K^0(t)|K^0\rangle|^2 = e^{-\Gamma t}$$

which satisfy the equation:

$$i\frac{\partial}{\partial t}|K^0(t)\rangle = \underbrace{\left(m - \frac{i}{2}\Gamma\right)}_{H}|K^0(t)\rangle$$

3. If K^0 can convert into $\overline{K^0}$ through second order mixing diagram, the time evolution of a neutral meson must include both K^0 and $\overline{K^0}$:



$$|K^0(t)\rangle = e^{-iHt}|K^0(t=0)\rangle = e^{-iHt}\frac{1}{\sqrt{2}}\left(|K_S^0\rangle + |K_L^0\rangle\right) =$$

$$= \frac{1}{\sqrt{2}}\left[e^{-i\left(m_S - \frac{i\Gamma_S}{2}\right)t}\,|K_S^0\rangle + e^{-i\left(m_L - \frac{i\Gamma_L}{2}\right)t}\,|K_L^0\rangle\right] = \ldots = \ldots = \ldots$$

so let's be more general: $|\psi(t)\rangle = a(t)|K^0\rangle + b(t)|\overline{K^0}\rangle$

Time evolution of neutral mesons

1. The eigenstates of effective Hamiltonian (weak) written in the form:

$$|P_1\rangle = p|P^0\rangle + q|\overline{P^0}\rangle$$

$$|P_2\rangle = p|P^0\rangle - q|\overline{P^0}\rangle$$

p and q are complex numbers satisfying: $|p|^2 + |q|^2 = 1$ (for K_1^0 and K_2^0 : $p = q = \frac{1}{\sqrt{2}}$)

2. Solving Schrödinger equation we see time evolution of the eigenstates:

$$|P_1(t)\rangle = |P_1\rangle\, e^{-i\left(m_1 - \frac{i\Gamma_1}{2}\right)t}$$

$$|P_2(t)\rangle = |P_2\rangle\, e^{-i\left(m_2 - \frac{i\Gamma_2}{2}\right)t}$$

These relations show that the original P^0 meson after some time can either convert to $\overline{P^0}$ or decay.

Time evolution of neutral mesons

9. Finally the time evolution of weak eigenstates as a combination of flavour eigenstates:

$$|P^0(t)\rangle = f_+(t)|P^0\rangle + \frac{q}{p} f_-(t)|\overline{P^0}\rangle$$

$$|\overline{P^0}(t)\rangle = f_+(t)|\overline{P^0}\rangle + \frac{p}{q} f_-(t)|P^0\rangle$$

$$f_\pm(t) = \frac{1}{2}\left[e^{-i(m_1-\frac{i}{2}\Gamma_1)t} \pm e^{-i(m_2-\frac{i}{2}\Gamma_2)t}\right]$$

solve this!

$$|f_\pm(t)|^2 = \frac{1}{4}\left[e^{-i\Gamma_1 t} + e^{-i\Gamma_2 t} \pm 2e^{-\bar{\Gamma}t}\cos(\Delta m t)\right]$$

$$\bar{\Gamma} = \frac{\Gamma_1 + \Gamma_2}{2}$$

interference term

10. The time evolution of mixing probabilities, i.e. the probability that having started the observation with a P^0 meson, after some time t we still have P^0 (or it has oscillated to $\overline{P^0}$):

$$P(P^0 \to P^0; \mathrm{t}) = |\langle P^0|P^0(t)\rangle|^2 = |f_+(t)|^2$$

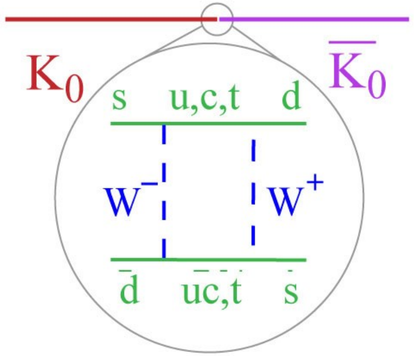
$$P(P^0 \to \overline{P^0}; \mathrm{t}) = |\langle\overline{P^0}|P^0(t)\rangle|^2 = \left|\frac{q}{p} f_-(t)\right|^2$$

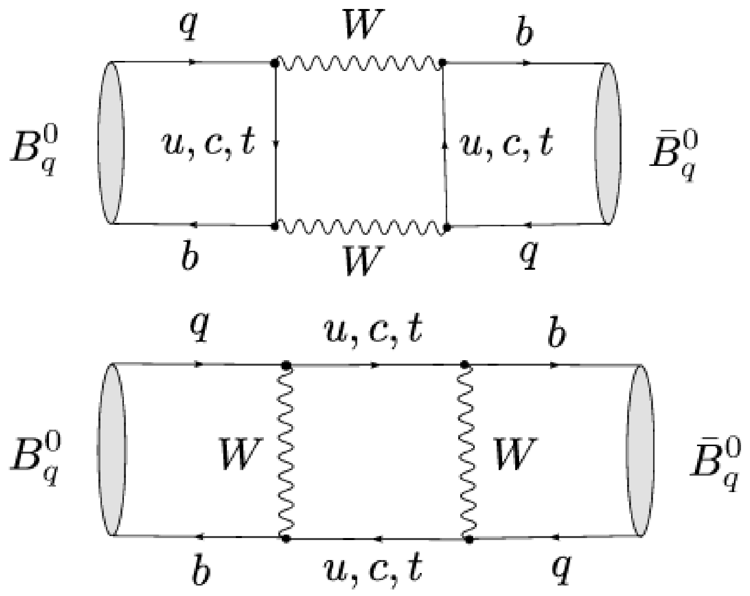
Let's look closer at the parameters of flavour oscillations:

CP violation in mixing

- Weak interactions makes possible the change of quark flavour.

This rule can do some magic transition from matter to antimatter:





- We found that having started the observation with a $\boldsymbol{P^0}$ meson, after some time $\boldsymbol{t}$ we can have $\overline{\boldsymbol{P^0}}$ ($\boldsymbol{P^0}$ has oscillated to $\overline{\boldsymbol{P^0}}$)!
- SM and V_{CKM} provide us with the parameters of oscillations