

$$\begin{aligned}
|K^0(t)\rangle &= \frac{1}{2} \frac{\sqrt{2(1+\epsilon)^2}}{1+\epsilon} (|K_S^0(t)\rangle + |K_L^0(t)\rangle) = \\
&= \frac{\sqrt{2(1+\epsilon)^2}}{2(1+\epsilon)} \left(e^{-im_S t - \frac{\gamma_S t}{2}} |K_S^0\rangle + e^{-im_L t - \frac{\gamma_L t}{2}} |K_L^0\rangle \right) = \\
&= \frac{\sqrt{2(1+\epsilon)^2}}{2(1+\epsilon)} \left(e^{-im_S t - \frac{\gamma_S t}{2}} \frac{1}{\sqrt{1+\epsilon^2}} \left\{ \frac{1+\epsilon}{2} |K^0\rangle - \frac{1-\epsilon}{2} |\bar{K}^0\rangle \right\} \right. \\
&\quad \left. + e^{-im_L t - \frac{\gamma_L t}{2}} \frac{1}{\sqrt{1+\epsilon^2}} \left\{ \frac{1+\epsilon}{2} |K^0\rangle + \frac{1-\epsilon}{2} |\bar{K}^0\rangle \right\} \right) \\
&= \frac{1}{2(1+\epsilon)} \left[(1+\epsilon) \left(e^{-im_S t - \frac{\gamma_S t}{2}} + e^{-im_L t - \frac{\gamma_L t}{2}} \right) |K^0\rangle \right. \\
&\quad \left. + (1-\epsilon) \left(e^{-im_L t - \frac{\gamma_L t}{2}} - e^{-im_S t - \frac{\gamma_S t}{2}} \right) |\bar{K}^0\rangle \right]
\end{aligned}$$

$$\begin{aligned}
|\langle K^0 | K^0(t) \rangle|^2 &= \frac{1}{4(1+\epsilon)^2} \left| \langle K^0 | (1+\epsilon) \left(e^{-im_S t - \frac{\gamma_S t}{2}} + e^{-im_L t - \frac{\gamma_L t}{2}} \right) |K^0\rangle \right. \\
&\quad \left. + \langle K^0 | (1-\epsilon) \left(e^{-im_L t - \frac{\gamma_L t}{2}} - e^{-im_S t - \frac{\gamma_S t}{2}} \right) |\bar{K}^0\rangle \right|^2 \\
&= \frac{1}{4(1+\epsilon)^2} \cdot (1+\epsilon)^2 \left(e^{-im_S t - \frac{\gamma_S t}{2}} + e^{-im_L t - \frac{\gamma_L t}{2}} \right)^2 \\
&= \frac{1}{4} e^{-\frac{t}{\tau_S}} + \frac{1}{4} e^{-\frac{t}{\tau_L}} + \frac{1}{2} e^{-\left(\frac{1}{\tau_S} + \frac{1}{\tau_L}\right)t} \cos(\Delta m t)
\end{aligned}$$

$$\langle \bar{K}^0 | \bar{K}^0(t) \rangle$$

$$|\bar{K}^0\rangle = \frac{1}{2} (K_S - K_L) \cdot \sqrt{2(1+|\epsilon|^2)} \cdot \frac{1}{1+\epsilon}$$

$$= \frac{1}{2} \left(\underbrace{e^{-im_S t - \frac{\gamma_S}{2} t}}_a |K_S\rangle - \underbrace{e^{-im_L t - \frac{\gamma_L}{2} t}}_b |K_L\rangle \right) \cdot \sqrt{2(1+|\epsilon|^2)} \cdot \frac{1}{1+\epsilon} =$$

$$= \frac{1}{2} \left[a \left[\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{1+|\epsilon|^2}} \left[(\epsilon+1) |K^0\rangle - (1-\epsilon) |\bar{K}^0\rangle \right] \right] - b \left[\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{1+|\epsilon|^2}} \left[(1+\epsilon) |K^0\rangle + (1-\epsilon) |\bar{K}^0\rangle \right] \right] \right]$$

$$= \frac{1}{2} \left[\frac{a(\epsilon+1)}{\sqrt{2} \cdot \sqrt{1+|\epsilon|^2}} - \frac{b(\epsilon+1)}{\sqrt{2} \cdot \sqrt{1+|\epsilon|^2}} \right] |K^0\rangle - \frac{1}{2} \left[\frac{a(1-\epsilon)}{\sqrt{2} \cdot \sqrt{1+|\epsilon|^2}} + \frac{b(1-\epsilon)}{\sqrt{2} \cdot \sqrt{1+|\epsilon|^2}} \right] |\bar{K}^0\rangle =$$

$$= \frac{\sqrt{2}(\epsilon+1)}{4\sqrt{1+|\epsilon|^2}} \begin{pmatrix} e^{-im_S t - \frac{\gamma_S}{2} t} & -e^{-im_L t - \frac{\gamma_L}{2} t} \\ -e^{-im_S t - \frac{\gamma_S}{2} t} & e^{-im_L t - \frac{\gamma_L}{2} t} \end{pmatrix} |K^0\rangle - \frac{\sqrt{2}(1-\epsilon)}{4\sqrt{1+|\epsilon|^2}} \begin{pmatrix} e^{-im_S t - \frac{\gamma_S}{2} t} & -e^{-im_L t - \frac{\gamma_L}{2} t} \\ -e^{-im_S t - \frac{\gamma_S}{2} t} & e^{-im_L t - \frac{\gamma_L}{2} t} \end{pmatrix} |\bar{K}^0\rangle$$

$$\langle \bar{K}^0 | \bar{K}^0(t) \rangle = -\frac{\sqrt{2}(1-\epsilon)}{4\sqrt{1+|\epsilon|^2}} \begin{pmatrix} e^{-im_S t - \frac{\gamma_S}{2} t} & -e^{-im_L t - \frac{\gamma_L}{2} t} \\ -e^{-im_S t - \frac{\gamma_S}{2} t} & e^{-im_L t - \frac{\gamma_L}{2} t} \end{pmatrix}$$

$$\langle K^0 | \bar{K}^0(t) \rangle = ?$$

$$|\bar{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1-\varepsilon} (|K_L\rangle - |K_S\rangle)$$

$$\begin{aligned} |\bar{K}^0(t)\rangle &= \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1-\varepsilon} (|K_L(t)\rangle - |K_S(t)\rangle) = \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1-\varepsilon} (f_L(t)|K_L\rangle - f_S|K_S\rangle) = \\ &= \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1-\varepsilon} \left(e^{-im_L t - \frac{\gamma_L}{2} t} |K_L\rangle - e^{-im_S t - \frac{\gamma_S}{2} t} |K_S\rangle \right) \end{aligned}$$

$$|K_L\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \left[(\varepsilon+1)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle \right]$$

$$|K_S\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \left[(\varepsilon+1)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle \right]$$

$$\langle K^0 | K^0 \rangle = 1.$$

$$\langle K^0 | \bar{K}^0 \rangle = 0.$$

$$\begin{aligned} \langle K^0 | \bar{K}^0(t) \rangle &= \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1-\varepsilon} \left[e^{-im_L t - \frac{\gamma_L}{2} t} \cdot \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \cdot (\varepsilon+1) - e^{-im_S t - \frac{\gamma_S}{2} t} \cdot \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \cdot (\varepsilon+1) \right] = \\ &= \frac{1+\varepsilon}{2(1-\varepsilon)} \left[e^{-im_L t - \frac{\gamma_L}{2} t} - e^{-im_S t - \frac{\gamma_S}{2} t} \right] \end{aligned}$$

$$3 = \langle (A^0)^2 | \psi \rangle$$

$$\langle (A^0)^2 | \psi \rangle = \frac{1}{3-1} \cdot \frac{\sqrt{3(1+1)}}{2} = \langle (A^0)^2 | \psi \rangle$$

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$$\left[\langle (A^0)^2 | (3-1) \rangle + \langle (A^0)^2 | (1+3) \rangle \right] \frac{1}{\sqrt{3(1+1)}} = \langle (A^0)^2 | \psi \rangle$$

$$\left[\langle (A^0)^2 | (3-1) \rangle - \langle (A^0)^2 | (1+3) \rangle \right] \frac{1}{\sqrt{3(1+1)}} = \langle (A^0)^2 | \psi \rangle$$

$$1 = \langle (A^0)^2 | \psi \rangle$$

$$0 = \langle (A^0)^2 | \psi \rangle$$

$$\left[\frac{1}{3+1} \cdot \frac{\sqrt{3(1+1)}}{2} \right] \frac{1}{\sqrt{3(1+1)}} = \langle (A^0)^2 | \psi \rangle$$

$$\left[\frac{1}{3+1} \cdot \frac{\sqrt{3(1+1)}}{2} \right] \frac{3+1}{\sqrt{3(1+1)}} = \langle (A^0)^2 | \psi \rangle$$

$$\langle \bar{K}^0 | K^0(t) \rangle = (*)$$

$$|K^0\rangle = (|K_S\rangle + |K_L\rangle) \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \frac{1}{1+\epsilon} \cdot \frac{1}{2}$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} \frac{1}{\sqrt{2}} \left\{ (1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle \right\}$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} \frac{1}{\sqrt{2}} \left\{ (1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle \right\}$$

$$K^0(t) = (|K_S(t)\rangle + |K_L(t)\rangle) \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \frac{1}{1-\epsilon} \cdot \frac{1}{2} =$$

$$= (f_S(t)|K_S\rangle + f_L(t)|K_L\rangle) \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \frac{1}{1-\epsilon} \cdot \frac{1}{2} =$$

$$= \left(f_S(t) \left[\frac{1}{\sqrt{2(1+|\epsilon|^2)}} \left\{ (1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle \right\} \right] + \right.$$

$$\left. + f_L(t) \left[\frac{1}{\sqrt{2(1+|\epsilon|^2)}} \left\{ (1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle \right\} \right] \right) \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \frac{1}{1-\epsilon} =$$

$$= \frac{1+\epsilon}{2(1-\epsilon)} (f_S(t) + f_L(t)) |K^0\rangle + \frac{1}{2} (f_L(t) - f_S(t)) |\bar{K}^0\rangle$$

$$\left. \begin{aligned} \langle \bar{K}^0 | K^0 \rangle &= 0 \\ \langle \bar{K}^0 | \bar{K}^0 \rangle &= 1 \end{aligned} \right\}$$

$$(*) = \frac{1}{2} \langle \bar{K}^0 | \bar{K}^0 \rangle (f_L(t) - f_S(t)) =$$

$$= \frac{1}{2} \left(e^{-im_L t} e^{-\frac{\gamma_L}{2} t} - e^{-im_S t} e^{-\frac{\gamma_S}{2} t} \right)$$