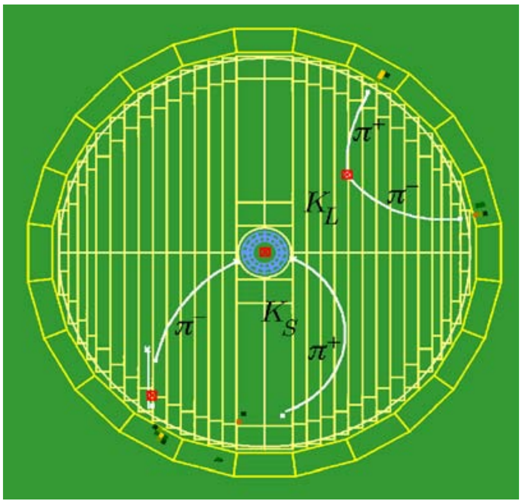
Particle Physics

Michaelmas Term 2011
Prof Mark Thomson



Handout 12 : The CKM Matrix and CP Violation

CP Violation in the Early Universe

- Very early in the universe might expect equal numbers of baryons and anti-baryons
- However, today the universe is matter dominated (no evidence for anti-galaxies, etc.)
- From "Big Bang Nucleosynthesis" obtain the matter/anti-matter asymmetry

$$\xi = \frac{n_B - n_{\overline{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

i.e. for every baryon in the universe today there are 10^9 photons

- How did this happen?

★ Early in the universe need to create a very small asymmetry between baryons and anti-baryons

e.g. for every 10^9 anti-baryons there were 10^9+1 baryons
baryons/anti-baryons annihilate ⇨
1 baryon + ~10^9 photons + no anti-baryons

★ To generate this initial asymmetry three conditions must be met (Sakharov, 1967):

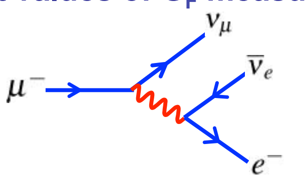
❶ "Baryon number violation", i.e. $n_B - n_{\overline{B}}$ is not constant
❷ "C and CP violation", if CP is conserved for a reaction which generates a net number of baryons over anti-baryons there would be a CP conjugate reaction generating a net number of anti-baryons
❸ "Departure from thermal equilibrium", in thermal equilibrium any baryon number violating process will be balanced by the inverse reaction

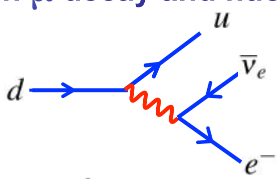
- CP Violation is an essential aspect of our understanding of the universe
- A natural question is whether the SM of particle physics can provide the necessary CP violation?
- There are two places in the SM where CP violation enters: the **PMNS matrix** and the **CKM matrix**
- **To date CP violation has been observed only in the quark sector**
- Because we are dealing with quarks, which are only observed as **bound states**, this is a fairly complicated subject. Here we will approach it in two steps:
  - i) Consider particle – anti-particle oscillations without CP violation
  - ii) Then discuss the effects of CP violation

★ Many features in common with neutrino oscillations – except that we will be considering the oscillations of decaying particles (i.e. mesons) !

The Weak Interaction of Quarks

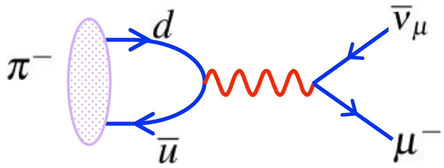
★ Slightly different values of G_F measured in μ decay and nuclear β decay:

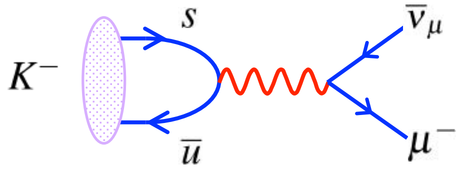




$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2} \qquad G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

★ In addition, certain hadronic decay modes are observed to be suppressed, e.g. compare $K^- \to \mu^- \bar{\nu}_\mu$ and $\pi^- \to \mu^- \bar{\nu}_\mu$. Kaon decay rate suppressed factor 20 compared to the expectation assuming a universal weak interaction for quarks.



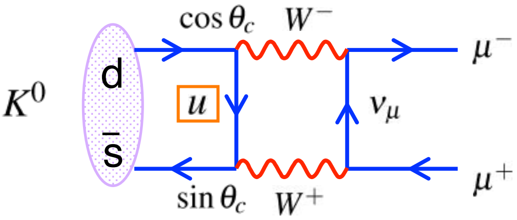


- Both observations explained by Cabibbo hypothesis (1963): weak eigenstates are different from mass eigenstates, i.e. weak interactions of quarks have same strength as for leptons but a u-quark couples to a linear combination of s and d

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

GIM Mechanism

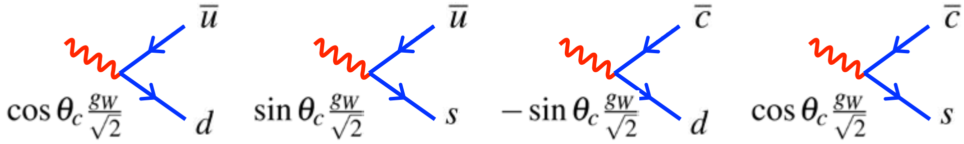
★ In the weak interaction have couplings between both ud and us which implies that neutral mesons can decay via box diagrams, e.g.



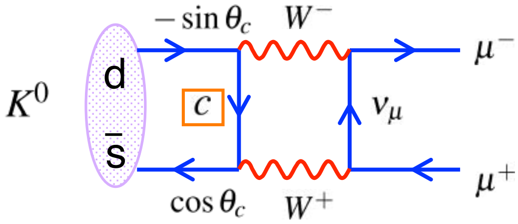
$$\boxed{M_1 \propto g_W^4 \cos\theta_c \sin\theta_c}$$

•Historically, the observed branching was **much smaller** than predicted

★ Led **Glashow**, **Illiopoulos** and **Maiani** to postulate existence of an extra quark - <u>**before**</u> discovery of charm quark in **1974**. Weak interaction couplings become



★ Gives another box diagram for $K^0 \to \mu^+\mu^-$



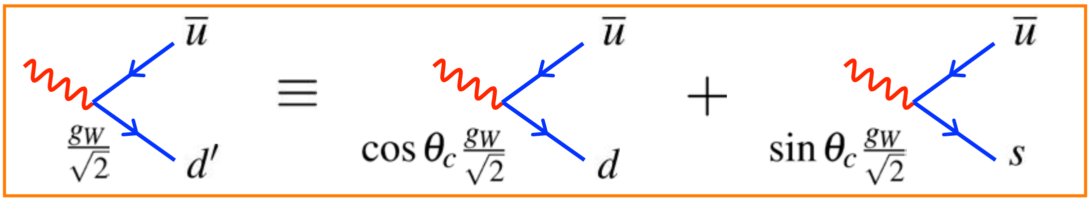
$$\boxed{M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c}$$

•Same final state so sum <u>amplitudes</u>

$$|M|^2 = |M_1 + M_2|^2 \approx 0$$

•Cancellation not exact because $m_u \neq m_c$

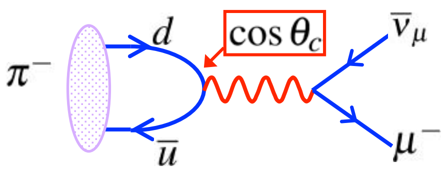
i.e. **weak interaction couples different generations of quarks**

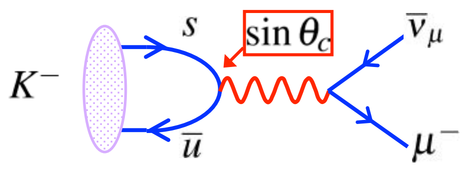


(The same is true for leptons e.g. $e^-\nu_1$, $e^-\nu_2$, $e^-\nu_3$ couplings – connect different generations)

★ Can explain the observations on the previous pages with $\theta_c = 13.1^\circ$

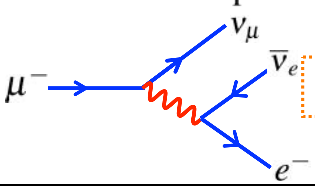
•Kaon decay suppressed by a factor of $\tan^2\theta_c \approx 0.05$ relative to pion decay



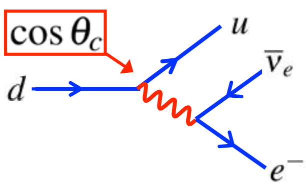


$$\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \cos^2\theta_c \qquad \Gamma(K^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \sin^2\theta_c$$

• Hence expect $G_F^\beta = G_F^\mu \cos\theta_c$



$\propto G_F.G_F$



$\propto G_F.(G_F\cos^2\theta_c)$

CKM Matrix

★ Extend ideas to three quark flavours (analogue of three flavour neutrino treatment)

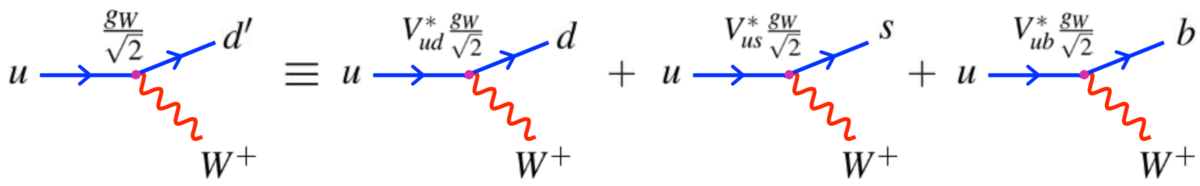
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

By convention CKM matrix defined as acting on quarks with charge $-\frac{1}{3}e$

Weak eigenstates | CKM Matrix | Mass Eigenstates

(Cabibbo, Kobayashi, Maskawa)

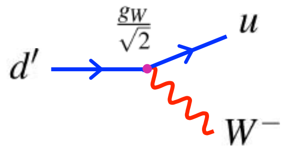
★ e.g. Weak eigenstate d' is produced in weak decay of an up quark:



- The CKM matrix elements V_{ij} are complex constants
- The CKM matrix is unitary
- The V_{ij} are not predicted by the SM – have to determined from experiment

Feynman Rules

- Depending on the order of the interaction, $u \to d$ or $d \to u$, the CKM matrix enters as either V_{ud} or V^*_{ud}
- Writing the interaction in terms of the WEAK eigenstates



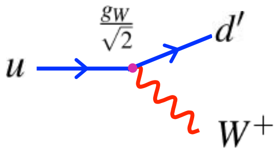
$$j_{d'u} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] d'$$

NOTE: ū is the adjoint spinor not the anti-up quark

- Giving the $\boxed{d \to u}$ weak current:

$$\boxed{j_{du} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] V_{ud}\, d}$$

- For $u \to d'$ the weak current is:



$$j_{ud'} = \bar{d}'\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u$$

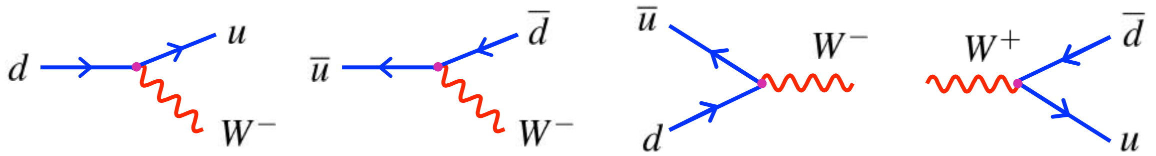
- In terms of the mass eigenstates $\bar{d}' = d'^{\dagger}\gamma^0 \to (V_{ud}d)^{\dagger}\gamma^0 = V^*_{ud}d^{\dagger}\gamma^0 = V^*_{ud}\bar{d}$

- Giving the $\boxed{u \to d}$ weak current:

$$\boxed{j_{ud} = \bar{d}V^*_{ud}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u}$$

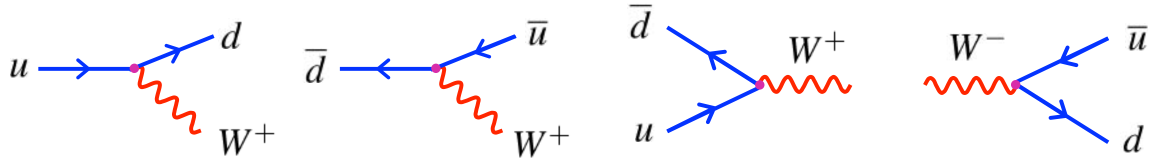
•Hence, when the charge $-\frac{1}{3}$ quark enters as the adjoint spinor, the complex conjugate of the CKM matrix is used

★ The vertex factor the following diagrams:



is $\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)}$

★ Whereas, the vertex factor for:



is $\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}^{*}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)}$

★ Experimentally (see Appendix I) determine

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

★ Currently little direct experimental information on V_{td}, V_{ts}, V_{tb}

★ Assuming unitarity of CKM matrix, e.g. $|V_{ub}|^2+|V_{cb}|^2+|V_{tb}|^2=1$ gives:

Cabibbo matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

Near diagonal – very different from PMNS

★ NOTE: within the SM, the charged current, $W^{\pm}$, weak interaction:
① Provides the only way to change flavour !
② only way to change from one generation of quarks or leptons to another !

★ However, the off-diagonal elements of the CKM matrix are relatively small.
• Weak interaction largest between quarks of the same generation.
• Coupling between first and third generation quarks is very small !

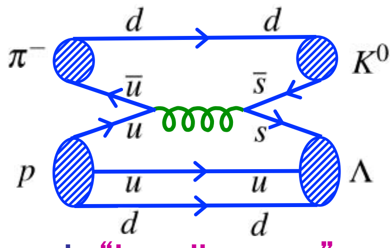
★ Just as for the PMNS matrix – the CKM matrix allows CP violation in the SM

The Neutral Kaon System

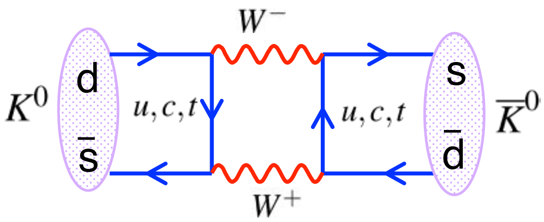
- **Neutral Kaons are produced copiously in strong interactions, e.g.**

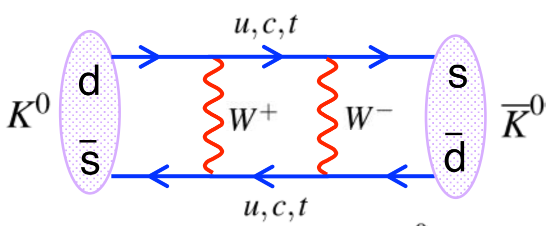
$$\pi^-(d\bar{u}) + p(uud) \to \Lambda(uds) + K^0(d\bar{s})$$

$$\pi^+(u\bar{d}) + p(uud) \to K^+(u\bar{s}) + \overline{K}^0(s\bar{d}) + p(uud)$$



- **Neutral Kaons decay via the weak interaction**
- **The Weak Interaction also allows mixing of neutral kaons via "box diagrams"**





- **This allows transitions between the strong eigenstates states** $K^0, \overline{K}^0$
- **Consequently, the neutral kaons propagate as eigenstates of the overall strong + weak interaction (Appendix II); i.e. as linear combinations of** $K^0, \overline{K}^0$
- **These neutral kaon states are called the "K-short"** K_S **and the "K-long"** K_L
- **These states have approximately the same mass** $m(K_S) \approx m(K_L) \approx 498\,\text{MeV}$
- **But very different lifetimes:** $\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\text{s}}$ $\boxed{\tau(K_L) = 0.5 \times 10^{-7}\,\text{s}}$

CP Eigenstates

★ **The** K_S **and** K_L **are closely related to eigenstates of the combined charge conjugation and parity operators: CP**

- **The strong eigenstates** $K^0(d\bar{s})$ **and** $\overline{K}^0(s\bar{d})$ **have** $J^P = 0^-$

with $$\boxed{\hat{P}|K^0\rangle = -|K^0\rangle, \quad \hat{P}|\overline{K}^0\rangle = -|\overline{K}^0\rangle}$$

- **The charge conjugation operator changes particle into anti-particle and *vice versa***

$$\hat{C}|K^0\rangle = \hat{C}|d\bar{s}\rangle = +|s\bar{d}\rangle = |\overline{K}^0\rangle$$

similarly $$\hat{C}|\overline{K}^0\rangle = |K^0\rangle$$

The + sign is purely conventional, could have used a - with no physical consequences

- **Consequently**

$$\boxed{\hat{C}\hat{P}|K^0\rangle = -|\overline{K}^0\rangle} \quad \boxed{\hat{C}\hat{P}|\overline{K}^0\rangle = -|K^0\rangle}$$

i.e. neither K^0 **or** $\overline{K}^0$ **are eigenstates of CP**

- **Form CP eigenstates from linear combinations:**

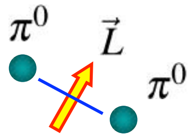
$$|K_1\rangle = \tfrac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle$$

$$|K_2\rangle = \tfrac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle$$

Decays of CP Eigenstates

- Neutral kaons often decay to pions (the lightest hadrons)
- The kaon masses are approximately **498 MeV** and the pion masses are approximately **140 MeV**. Hence neutral kaons can decay to either 2 or 3 pions

Decays to Two Pions:



★ $K^0 \to \pi^0\pi^0 \qquad J^P: \quad 0^- \to 0^- + 0^-$

- Conservation of angular momentum ➡ $\vec{L} = 0$

$$\Rightarrow \hat{P}(\pi^0\pi^0) = -1.-1.(-1)^L = +1$$

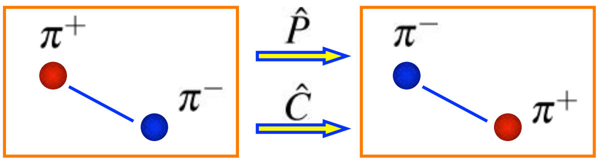
- The $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ is an eigenstate of $\hat{C}$

$$C(\pi^0\pi^0) = C\pi^0.C\pi^0 = +1.+1 = +1$$

$$\Rightarrow \boxed{CP(\pi^0\pi^0) = +1}$$

★ $K^0 \to \pi^+\pi^-$ as before $\hat{P}(\pi^+\pi^-) = +1$

★ Here the C and P operations have the identical effect



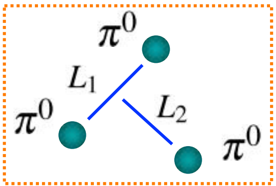
Hence the combined effect of $\hat{C}\hat{P}$ is to leave the system unchanged

$$\boxed{\hat{C}\hat{P}(\pi^+\pi^-) = +1}$$

Neutral kaon decays to two pions occur in CP even (i.e. +1) eigenstates

Decays to Three Pions:

★ $K^0 \to \pi^0\pi^0\pi^0 \qquad J^P: \quad 0^- \to 0^- + 0^- + 0^-$



- Conservation of angular momentum:

Remember L is magnitude of angular momentum vector

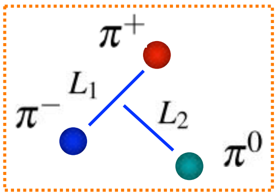
$$L_1 \oplus L_2 = 0 \quad \Rightarrow \quad L_1 = L_2$$

$$P(\pi^0\pi^0\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^0\pi^0\pi^0) = +1.+1.+1$$

$$\Rightarrow \boxed{CP(\pi^0\pi^0\pi^0) = -1}$$

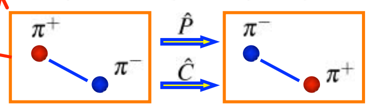
★ $K^0 \to \pi^+\pi^-\pi^0$



- Again $L_1 = L_2$

$$P(\pi^+\pi^-\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^+\pi^-\pi^0) = +1.C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^{L_1}$$



Hence: $\boxed{CP(\pi^+\pi^-\pi^0) = -1.(-1)^{L_1}}$

- The small amount of energy available in the decay, $m(K) - 3m(\pi) \approx 70\,\text{MeV}$ means that the L>0 decays are strongly suppressed by the angular momentum barrier effects (recall QM tunnelling in alpha decay)

Neutral kaon decays to three pions occur in CP odd (i.e. -1) eigenstates

★ <u>If</u> CP were conserved in the Weak decays of neutral kaons, would expect decays to pions to occur from states of definite CP (i.e. the CP eigenstates K_1 , K_2)

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle \qquad K_1 \to \pi\pi \qquad \textbf{CP EVEN}$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle \qquad K_2 \to \pi\pi\pi \qquad \textbf{CP ODD}$$

★ Expect lifetimes of CP eigenstates to be very different
- For two pion decay energy available: $m_K - 2m_\pi \approx 220\,\mathrm{MeV}$
- For three pion decay energy available: $m_K - 3m_\pi \approx 80\,\mathrm{MeV}$

★ Expect decays to two pions to be more rapid than decays to three pions due to increased phase space

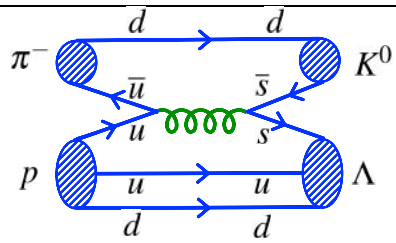
★ This is exactly what is observed: a short-lived state "K-short" which decays to (mainly) to two pions and a long-lived state "K-long" which decays to three pions

★ In the absence of CP violation we can identify

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \text{with decays:} \qquad K_S \to \pi\pi$$

$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \text{with decays:} \qquad K_L \to \pi\pi\pi$$

Neutral Kaon Decays to pions

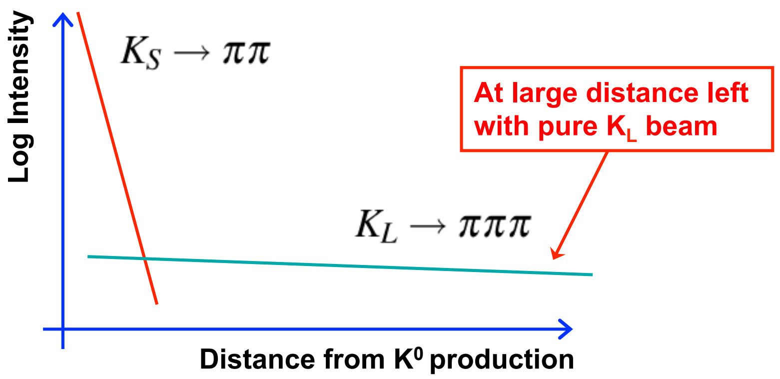


- Consider the decays of a beam of K^0
- The decays to pions occur in states of definite CP
- If CP is conserved in the decay, need to express K^0 in terms of K_S and K_L

$$|K_0\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$$

- Hence from the point of view of decays to pions, a K^0 beam is a linear combination of CP eigenstates:

  a rapidly decaying CP-even component and a long-lived CP-odd component
- Therefore, expect to see predominantly two-pion decays near start of beam and predominantly three pion decays further downstream



★To see how this works algebraically:

•Suppose at time t=0 make a beam of pure K^0

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$$

•Put in the time dependence of wave-function

$$|K_S(t)\rangle = |K_S\rangle e^{-im_S t - \Gamma_S t/2}$$

K_S mass: m_S
K_S decay rate: $\Gamma_S = 1/\tau_S$

NOTE the term $e^{-\Gamma_S t/2}$ ensures the K_S probability density decays exponentially

i.e. $|\psi_S|^2 = \langle K_S(t)|K_S(t)\rangle = e^{-\Gamma_S t} = e^{-t/\tau_S}$

•Hence wave-function evolves as

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|K_S\rangle e^{-(im_S+\frac{\Gamma_S}{2})t} + |K_L\rangle e^{-(im_L+\frac{\Gamma_L}{2})t}\right]$$

•Writing $\theta_S(t) = e^{-(im_S+\frac{\Gamma_S}{2})t}$ and $\theta_L(t) = e^{-(im_L+\frac{\Gamma_L}{2})t}$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle)$$

•The **decay rate to two pions** for a state which was produced as K^0:

$$\Gamma(K^0_{t=0} \to \pi\pi) \propto |\langle K_S|\psi(t)\rangle|^2 \propto |\theta_S(t)|^2 = e^{-\Gamma_S t} = e^{-t/\tau_S}$$

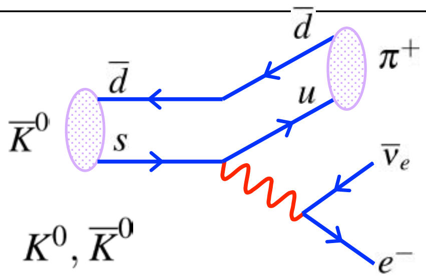
which is as anticipated, i.e. decays of the short lifetime component **K_S**

Neutral Kaon Decays to Leptons

•Neutral kaons can also decay to leptons

$\overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$ $\quad\overline{K}^0 \to \pi^+ \mu^- \overline{\nu}_\mu$

$K^0 \to \pi^- e^+ \nu_e$ $\quad K^0 \to \pi^- \mu^+ \nu_\mu$



•**Note:** the final states are not CP eigenstates which is why we express these decays in terms of $K^0, \overline{K}^0$

• Neutral kaons propagate as combined eigenstates of weak + strong interaction i.e. the K_S, K_L. The main decay modes/branching fractions are:

K_S	$\to$	$\pi^+\pi^-$	$BR = 69.2\%$
	$\to$	$\pi^0\pi^0$	$BR = 30.7\%$
	$\to$	$\pi^- e^+ \nu_e$	$BR = 0.03\%$
	$\to$	$\pi^+ e^- \overline{\nu}_e$	$BR = 0.03\%$
	$\to$	$\pi^- \mu^+ \nu_\mu$	$BR = 0.02\%$
	$\to$	$\pi^+ \mu^- \overline{\nu}_\mu$	$BR = 0.02\%$

K_L	$\to$	$\pi^+\pi^-\pi^0$	$BR = 12.6\%$
	$\to$	$\pi^0\pi^0\pi^0$	$BR = 19.6\%$
	$\to$	$\pi^- e^+ \nu_e$	$BR = 20.2\%$
	$\to$	$\pi^+ e^- \overline{\nu}_e$	$BR = 20.2\%$
	$\to$	$\pi^- \mu^+ \nu_\mu$	$BR = 13.5\%$
	$\to$	$\pi^+ \mu^- \overline{\nu}_\mu$	$BR = 13.5\%$

•Leptonic decays are more likely for the K-long because the three pion decay modes have a lower decay rate than the two pion modes of the K-short

Strangeness Oscillations (neglecting CP violation)

- The "semi-leptonic" decay rate to $\pi^- e^+ \nu_e$ occurs from the K^0 state. Hence to calculate the expected decay rate, need to know the K^0 component of the wave-function. For example, for a beam which was initially K^0 we have (1)

$$|\psi(t)\rangle = \tfrac{1}{\sqrt{2}}(\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle)$$

- Writing K_S, K_L in terms of $K^0, \overline{K}^0$

$$\begin{aligned} |\psi(t)\rangle &= \tfrac{1}{2}\left[\theta_S(t)(|K^0\rangle - |\overline{K}^0\rangle) + \theta_L(t)(|K^0\rangle + |\overline{K}^0\rangle)\right] \\ &= \tfrac{1}{2}(\theta_S + \theta_L)|K^0\rangle + \tfrac{1}{2}(\theta_L - \theta_S)|\overline{K}^0\rangle \end{aligned}$$

- Because $\theta_S(t) \neq \theta_L(t)$ a state that was initially a K^0 evolves with time into a mixture of K^0 and $\overline{K}^0$ - "strangeness oscillations"

- The K^0 intensity (i.e. K^0 fraction):

$$\Gamma(K^0_{t=0} \to K^0) = |\langle K^0|\psi(t)\rangle|^2 = \tfrac{1}{4}|\theta_S + \theta_L|^2 \qquad (2)$$

- Similarly $$\Gamma(K^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0|\psi(t)\rangle|^2 = \tfrac{1}{4}|\theta_S - \theta_L|^2 \qquad (3)$$

- Using the identity $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re(z_1 z_2^*)$

$$\begin{aligned} |\theta_S \pm \theta_L|^2 &= |e^{-(im_S + \frac{1}{2}\Gamma_S)t} \pm e^{-(im_L + \frac{1}{2}\Gamma_L)t}|^2 \\ &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2\Re\{e^{-im_S t}e^{-\frac{1}{2}\Gamma_S t}.e^{+im_L t}e^{-\frac{1}{2}\Gamma_L t}\} \\ &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\Re\{e^{-i(m_S - m_L)t}\} \\ &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos(m_S - m_L)t \\ &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\Delta m t \end{aligned}$$

- Oscillations between neutral kaon states with frequency given by the mass splitting $$\Delta m = m(K_L) - m(K_S)$$

- Reminiscent of neutrino oscillations ! Only this time we have decaying states.
- Using equations (2) and (3):

$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right] \qquad (4)$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right] \qquad (5)$$

• **Experimentally we find:** $\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\mathrm{s}}$ $\boxed{\tau(K_L) = 0.5 \times 10^{-7}\,\mathrm{s}}$

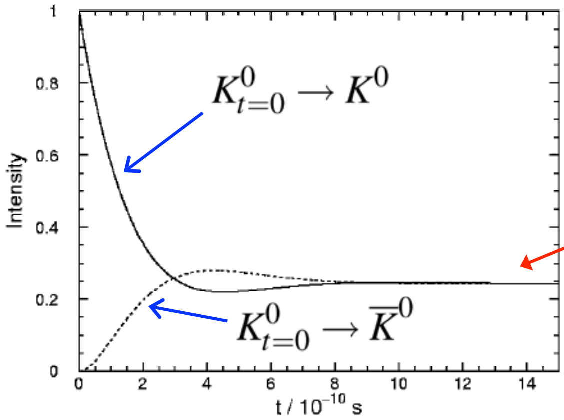
and $\boxed{\Delta m = (3.506 \pm 0.006) \times 10^{-15}\,\mathrm{GeV}}$

i.e. the K-long mass is greater than the K-short by 1 part in 10^{16}

• **The mass difference corresponds to an oscillation period of**

$$T_{osc} = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9}\,\mathrm{s}$$

• **The oscillation period is relatively long compared to the K_S lifetime and consequently, do not observe very pronounced oscillations**

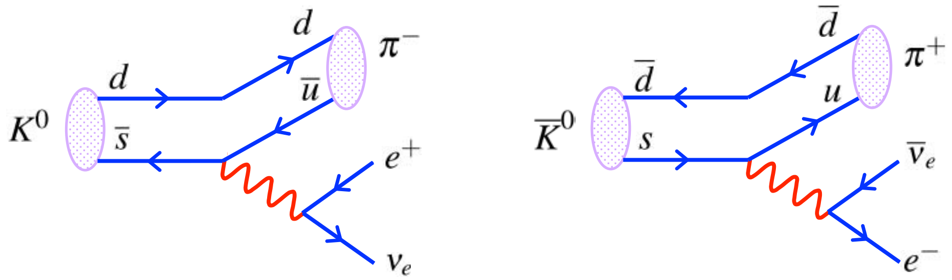


$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

After a few K_S lifetimes, left with a pure K_L beam which is half K^0 and half $\overline{K}^0$

★ **Strangeness oscillations can be studied by looking at semi-leptonic decays**



★ **The charge of the observed pion (or lepton) tags the decay as from either a $\overline{K}^0$ or K^0 because**

$$K^0 \to \pi^- e^+ \nu_e \qquad \overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$$

but

$$\overline{K}^0 \not\to \pi^- e^+ \nu_e \qquad K^0 \not\to \pi^+ e^- \overline{\nu}_e$$

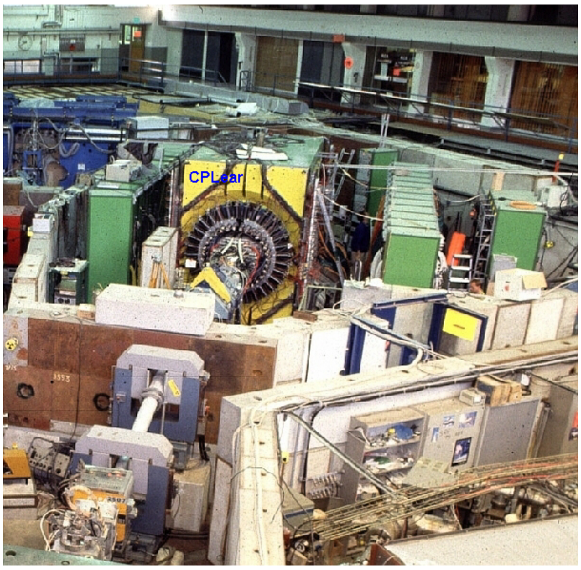
NOT ALLOWED (see Question 23)

•**So for an initial K^0 beam, observe the decays to both charge combinations:**

$$K^0_{t=0} \to K^0 \hookrightarrow \pi^- e^+ \nu_e \qquad K^0_{t=0} \to \overline{K}^0 \hookrightarrow \pi^+ e^- \overline{\nu}_e$$

which provides a way of measuring strangeness oscillations

The CPLEAR Experiment

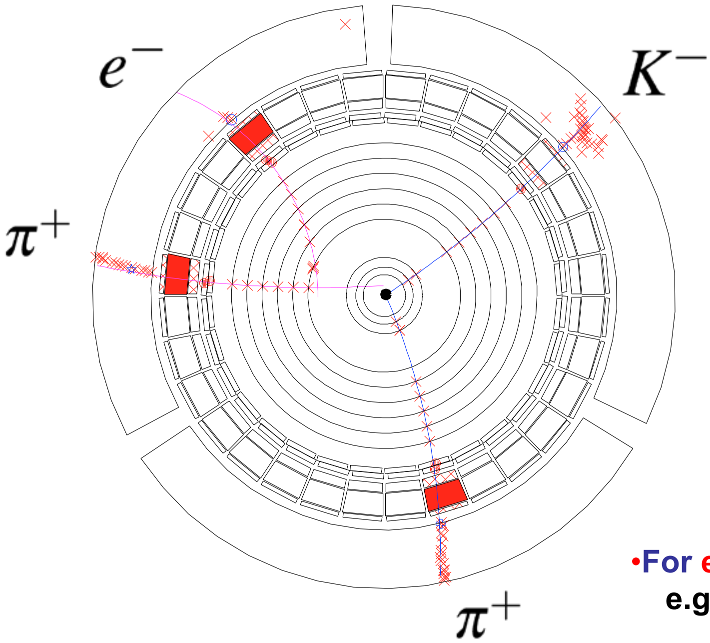


- CERN : 1990-1996
- Used a low energy anti-proton beam
- Neutral kaons produced in reactions

$$\bar{p}p \rightarrow K^-\pi^+K^0$$
$$\bar{p}p \rightarrow K^+\pi^-\overline{K}^0 \qquad \text{(Question 24)}$$

- Low energy, so particles produced almost at rest
- Observe production process and decay in the same detector
- Charge of $K^{\pm}\pi^{\mp}$ in the production process tags the initial neutral kaon as either K^0 or $\overline{K}^0$
- Charge of decay products tags the decay as either as being either K^0 or $\overline{K}^0$
- Provides a direct probe of strangeness oscillations

An example of a CPLEAR event



$$K^-(s\bar{u})$$
$$K^0(d\bar{s})$$
$$\overline{K}^0(s\bar{d})$$

Production:

$$\bar{p}p \rightarrow \boxed{K^-\pi^+}K^0$$

Mixing

Decay:

$$\overline{K}^0 \rightarrow \boxed{\pi^+e^-}\bar{\nu}_e$$

- For each event know initial wave-function, e.g. here: $|\psi(t=0)\rangle = |K^0\rangle$

•Can measure decay rates as a function of time for all combinations:

e.g. $R^{+} = \Gamma(K^0_{t=0} \to \pi^- e^+ \overline{\nu}_e) \propto \Gamma(K^0_{t=0} \to K^0)$

•From equations (4), (5) and similar relations:

$$R_+ \equiv \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e) = N_{\pi e \nu}\tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$R_- \equiv \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) = N_{\pi e \nu}\tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\overline{R}_- \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) = N_{\pi e \nu}\tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\overline{R}_+ \equiv \Gamma(\overline{K}^0_{t=0} \to \pi^- e^+ \nu_e) = N_{\pi e \nu}\tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

where $N_{\pi e \nu}$ is some overall normalisation factor

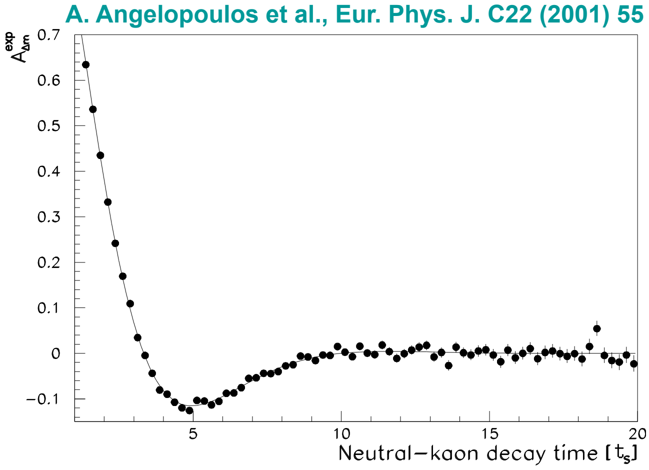
•Express measurements as an "asymmetry" to remove dependence on $N_{\pi e \nu}$

$$A_{\Delta m} = \frac{(R_+ + \overline{R}_-) - (R_- + \overline{R}_+)}{(R_+ + \overline{R}_-) + (R_- + \overline{R}_+)}$$

•Using the above expressions for R_+ etc., obtain

$$A_{\Delta m} = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

A. Angelopoulos et al., Eur. Phys. J. C22 (2001) 55



★ Points show the data

★ The line shows the theoretical prediction for the value of Δm most consistent with the CPLEAR data:

$$\boxed{\Delta m = 3.485 \times 10^{-15}\,\mathrm{GeV}}$$

•The sign of Δm is not determined here but is known from other experiments

• When the CPLEAR results are combined with experiments at FermiLab obtain:

$$\boxed{\Delta m = m(K_L) - m(K_S) = (3.506 \pm 0.006) \times 10^{-15}\,\mathrm{GeV}}$$

CP Violation in the Kaon System

★ So far we have ignored CP violation in the neutral kaon system
★ Identified the K-short as the CP-even state and the K-long as the CP-odd state

$$|K_S\rangle = |K_1\rangle \equiv \tfrac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \text{with decays:} \quad K_S \to \pi\pi \qquad \boxed{CP = +1}$$

$$|K_L\rangle = |K_2\rangle \equiv \tfrac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \text{with decays:} \quad K_L \to \pi\pi\pi \qquad \boxed{CP = -1}$$

★ At a long distance from the production point a beam of neutral kaons will be 100% K-long (the K-short component will have decayed away). Hence, if CP is conserved, would expect to see only three-pion decays.

★ In 1964 Fitch & Cronin (joint Nobel prize) observed 45 $K_L \to \pi^+\pi^-$ decays in a sample of 22700 kaon decays a long distance from the production point

⇨ **Weak interactions violate CP**

•CP is violated in hadronic weak interactions, but only at the level of 2 parts in 1000

K_L to pion BRs:

$K_L \to$	$\pi^+\pi^-\pi^0$	$BR = 12.6\%$	$CP = -1$
$\to$	$\pi^0\pi^0\pi^0$	$BR = 19.6\%$	$CP = -1$
$\to$	$\pi^+\pi^-$	$BR = 0.20\%$	$CP = +1$
$\to$	$\pi^0\pi^0$	$BR = 0.08\%$	$CP = +1$

★ Two possible explanations of CP violation in the kaon system:

i) The K_S and K_L do not correspond exactly to the CP eigenstates K_1 and K_2

$$\boxed{|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_1\rangle + \varepsilon|K_2\rangle]} \qquad \boxed{|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle]}$$

with $|\varepsilon| \sim 2\times 10^{-3}$

•In this case the observation of $K_L \to \pi\pi$ is accounted for by:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle]$$

$\varepsilon|K_1\rangle \to \pi\pi$ CP = +1

$|K_2\rangle \to \pi\pi\pi$ CP = -1

ii) and/or CP is violated in the decay

$|K_L\rangle = |K_2\rangle$ CP = -1

$\to \pi\pi\pi$ CP = -1

$\to \pi\pi$ CP = +1

Parameterised by ε'

★ Experimentally both known to contribute to the mechanism for CP violation in the kaon system but i) dominates: $\varepsilon'/\varepsilon = (1.7 \pm 0.3)\times 10^{-3}$ { NA48 (CERN), KTeV (FermiLab)

★ The dominant mechanism is discussed in examinable Appendix III

CP Violation in Semi-leptonic decays

★ **If observe a neutral kaon beam a long time after production (i.e. a large distances) it will consist of a pure K_L component**

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K_0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

$(1+\varepsilon)|K_0\rangle \longrightarrow \pi^+e^-\overline{\nu}_e$, $\quad (1-\varepsilon)|\overline{K}^0\rangle \longrightarrow \pi^-e^+\nu_e$

★ **Decays to $\pi^-e^+\nu_e$ must come from the $\overline{K}^0$ component, and decays to $\pi^+e^-\overline{\nu}_e$ must come from the K^0 component**

$$\Gamma(K_L \to \pi^+e^-\overline{\nu}_e) \propto |\langle\overline{K}^0|K_L\rangle|^2 \propto |1-\varepsilon|^2 \approx 1-2\Re\{\varepsilon\}$$

$$\Gamma(K_L \to \pi^-e^+\nu_e) \propto |\langle K^0|K_L\rangle|^2 \propto |1+\varepsilon|^2 \approx 1+2\Re\{\varepsilon\}$$

★ **Results in a small difference in decay rates: the decay to $\pi^-e^+\nu_e$ is 0.7 % more likely than the decay to $\pi^+e^-\overline{\nu}_e$**

- **This difference has been observed and thus provides the first direct evidence for an absolute difference between matter and anti-matter.**

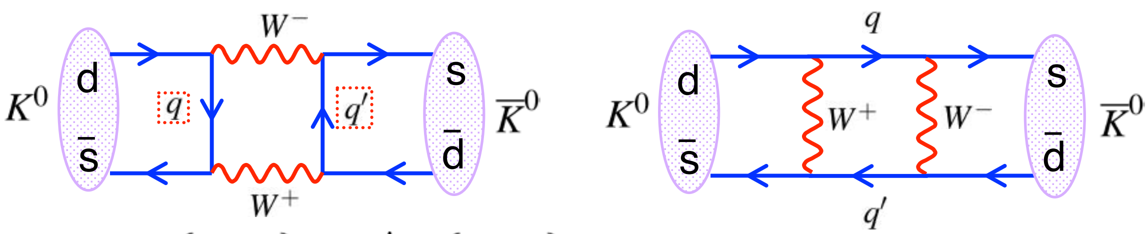
★ **It also provides an unambiguous definition of matter which could, for example, be transmitted to aliens in a distant galaxy**

> **"The electrons in our atoms have the same charge as those emitted least often in the decays of the long-lived neutral kaon"**

CP Violation and the CKM Matrix

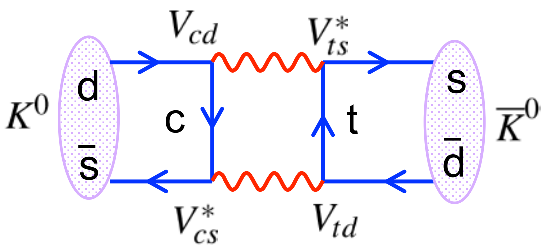
★ **How can we explain $\Gamma(\overline{K}^0_{t=0} \to K^0) \neq \Gamma(K^0_{t=0} \to \overline{K}^0)$ in terms of the CKM matrix ?**

★ **Consider the box diagrams responsible for mixing, i.e.**



where $q = \{u,c,t\}, \quad q' = \{u,c,t\}$

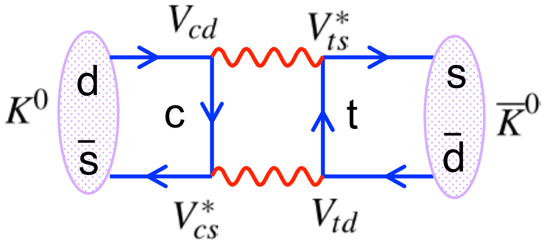
★ **Have to sum over all possible quark exchanges in the box. For simplicity consider just one diagram**



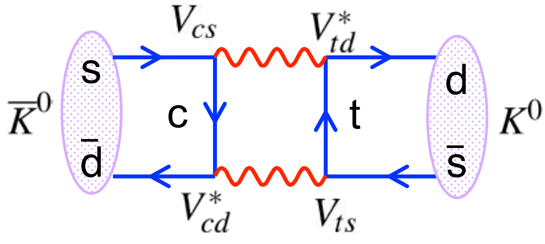
$$M_{fi} \propto A_{ct} V_{cd} V^*_{cs} V_{td} V^*_{ts}$$

A constant related to integrating over virtual momenta

★ Compare the equivalent box diagrams for $K^0 \rightarrow \overline{K}^0$ and $\overline{K}^0 \rightarrow K^0$



$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$$



$$M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$$

★ Therefore difference in rates

$$\Gamma(K^0 \rightarrow \overline{K}^0) - \Gamma(\overline{K}^0 \rightarrow K^0) \propto M_{fi} - M_{fi}^* = 2\Im\{M_{fi}\}$$

★ Hence the rates can only be different if the CKM matrix has imaginary component

$$|\varepsilon| \propto \Im\{M_{fi}\}$$

★ A more formal derivation is given in **Appendix IV**

★ In the kaon system we can show (question 25)

$$|\varepsilon| \propto A_{ut}.\Im\{V_{ud}V_{us}^*V_{td}V_{ts}^*\} + A_{ct}.\Im\{V_{cd}V_{cs}^*V_{td}V_{ts}^*\} + A_{tt}.\Im\{V_{td}V_{ts}^*V_{td}V_{ts}^*\}$$

Shows that CP violation is related to the imaginary parts of the CKM matrix

Summary

★ The weak interactions of quarks are described by the **CKM** matrix
★ Similar structure to the lepton sector, although unlike the **PMNS** matrix, the **CKM** matrix is nearly diagonal
★ CP violation enters through via a complex phase in the **CKM** matrix
★ A great deal of experimental evidence for CP violation in the weak interactions of quarks
★ CP violation is needed to explain matter – anti-matter asymmetry in the Universe
★ HOWEVER, CP violation in the SM is **not sufficient** to explain the matter – anti-matter asymmetry. There is probably another mechanism.

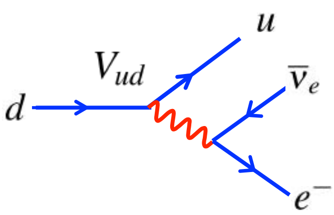
Appendix I: Determination of the CKM Matrix

Non-examinable

- The experimental determination of the **CKM matrix** elements comes mainly from measurements of leptonic decays (the leptonic part is well understood).
- It is easy to produce/observe meson decays, however theoretical uncertainties associated with the decays of bound states often limits the precision
- Contrast this with the measurements of the **PMNS matrix**, where there are few theoretical uncertainties and the experimental difficulties in dealing with neutrinos limits the precision.

❶ **$|V_{ud}|$** **from nuclear beta decay**

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



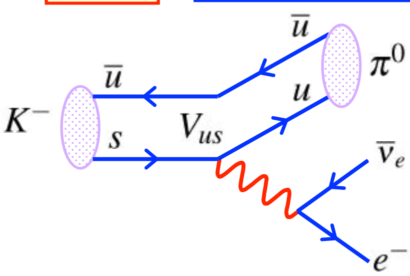
Super-allowed $0^+ \to 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$\boxed{|V_{ud}| = 0.97377 \pm 0.00027} \qquad (\approx \cos\theta_c)$$

❷ **$|V_{us}|$** **from semi-leptonic kaon decays**

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



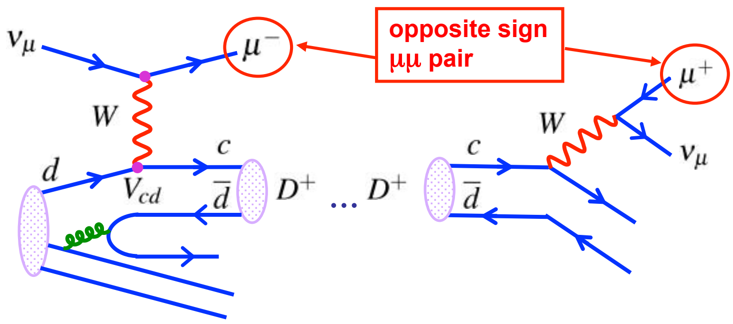
$$\Gamma \propto |V_{us}|^2$$

$$\boxed{|V_{us}| = 0.2257 \pm 0.0021} \qquad (\approx \sin\theta_c)$$

❸ **$|V_{cd}|$** **from neutrino scattering** $\nu_\mu + N \to \mu^+\mu^- X$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\bar{d})$ meson



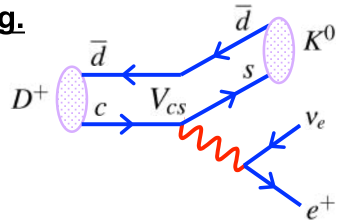
$$\text{Rate} \propto |V_{cd}|^2 \underbrace{\text{Br}(D^+ \to X\mu^+\nu_\mu)}_{\text{Measured in various collider experiments}}$$

$$\Rightarrow \quad \boxed{|V_{cd}| = 0.230 \pm 0.011}$$

4 **$|V_{cs}|$** **from semi-leptonic charmed meson decays**

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{cs}|^2$$

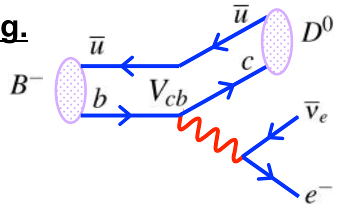
•**Precision limited by theoretical uncertainties**

$$\boxed{|V_{cs}| = 0.957 \pm \underbrace{0.017}_{\text{experimental error}} \pm \underbrace{0.093}_{\text{theory uncertainty}}}$$

5 **$|V_{cb}|$** **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



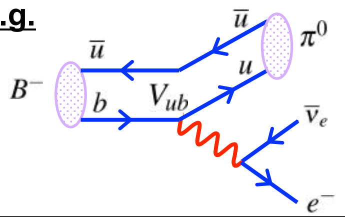
$$\Gamma \propto |V_{cb}|^2$$

$$\boxed{|V_{cb}| = 0.0416 \pm 0.0006}$$

6 **$|V_{ub}|$** **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{ub}|^2$$

$$\boxed{|V_{ub}| = 0.0043 \pm 0.0003}$$

Appendix II: Particle – Anti-Particle Mixing

Non-examinable

•**The wave-function for a single particle with lifetime $\tau = 1/\Gamma$ evolves with time as:**

$$\psi(t) = Ne^{-\Gamma t/2}e^{-iMt}$$

which gives the appropriate exponential decay of

$$\langle\psi(t)|\psi(t)\rangle = \langle\psi(0)|\psi(0)\rangle e^{-t/\tau}$$

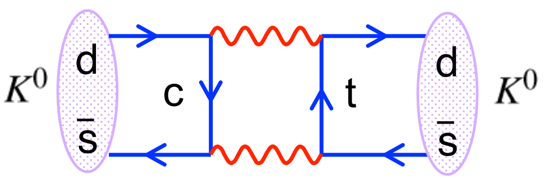
•**The wave-function satisfies the time-dependent wave equation:**

$$\hat{H}|\psi(t)\rangle = \boxed{(M - \tfrac{1}{2}i\Gamma)}|\psi(t)\rangle = i\frac{\partial}{\partial t}|\psi(t)\rangle \qquad \textbf{(A1)}$$

•**For a bound state such as a K^0 the mass term includes the "mass" from the weak interaction "potential" $\hat{H}_{\text{weak}}$**

$$M = m_{K^0} + \langle K^0|\hat{H}_{\text{weak}}|K^0\rangle + \sum_j \frac{|\langle K^0|\hat{H}_{\text{weak}}|j\rangle|^2}{m_{K^0} - E_j}$$

Sum over intermediate states j



The third term is the 2nd order term in the perturbation expansion corresponding to box diagrams resulting in $K^0 \to \bar{K}^0$

- The total decay rate is the sum over all possible decays $K^0 \to f$

$$\Gamma = 2\pi \sum_f |\langle f|\hat{H}_{weak}|K^0\rangle|^2 \rho_F$$

Density of final states

★ Because there are also diagrams which allow $K^0 \leftrightarrow \overline{K}^0$ mixing need to consider the time evolution of a mixed stated

$$\psi(t) = a(t)K^0 + b(t)\overline{K}^0 \qquad \text{(A2)}$$

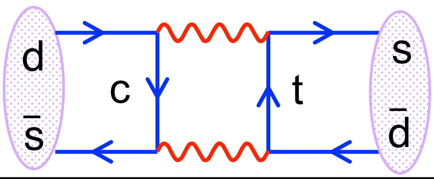
★ The time dependent wave-equation of (A1) becomes

$$\begin{pmatrix} M_{11} - \frac{1}{2}i\Gamma_{11} & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{21} - \frac{1}{2}i\Gamma_{21} & M_{22} - \frac{1}{2}i\Gamma_{22} \end{pmatrix} \begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix} \qquad \text{(A3)}$$

the diagonal terms are as before, and the off-diagonal terms are due to mixing.

$$M_{11} = m_{K^0} + \langle K^0|\hat{H}_{\text{weak}}|K^0\rangle + \sum_n \frac{|\langle K^0|\hat{H}_{\text{weak}}|K^0\rangle|^2}{m_{K^0} - E_n}$$

$$M_{12} = \sum_j \frac{\langle K^0|\hat{H}_{\text{weak}}|j\rangle^* \langle j|\hat{H}_{\text{weak}}|\overline{K}^0\rangle}{m_{K^0} - E_j}$$

K^0


$\overline{K}^0$

- The off-diagonal decay terms include the effects of interference between decays to a common final state

$$\Gamma_{12} = 2\pi \sum_f \langle f|\hat{H}_{weak}|K^0\rangle^* \langle f|\hat{H}_{weak}|\overline{K}^0\rangle \rho_F$$

- In terms of the time dependent coefficients for the kaon states, (A3) becomes

$$[\mathbf{M} - i\tfrac{1}{2}\Gamma]\begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix}$$

where the Hamiltonian can be written:

$$\mathbf{H} = \mathbf{M} - i\tfrac{1}{2}\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{1}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

- Both the mass and decay matrices represent observable quantities and are Hermitian

$$M_{11} = M_{11}^*, \quad M_{22} = M_{22}^*, \quad M_{12} = M_{21}^*$$

$$\Gamma_{11} = \Gamma_{11}^*, \quad \Gamma_{22} = \Gamma_{22}^*, \quad \Gamma_{12} = \Gamma_{21}^*$$

- Furthermore, if CPT is conserved then the masses and decay rates of the $\overline{K}^0$ and K^0 are identical:

$$M_{11} = M_{22} = M; \qquad \Gamma_{11} = \Gamma_{22} = \Gamma$$

•Hence the time evolution of the system can be written:

$$\boxed{\begin{pmatrix} M-\frac{1}{2}i\Gamma & M_{12}-\frac{1}{2}i\Gamma_{12} \\ M_{12}^*-\frac{1}{2}i\Gamma_{12}^* & M-\frac{1}{2}i\Gamma \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix}} \qquad \textbf{(A4)}$$

•To solve the coupled differential equations for a(t) and b(t), first find the eigenstates of the Hamiltonian (the K_L and K_S) and then transform into this basis. The eigenvalue equation is:

$$\begin{pmatrix} M-\frac{1}{2}i\Gamma & M_{12}-\frac{1}{2}i\Gamma_{12} \\ M_{12}^*-\frac{1}{2}i\Gamma_{12}^* & M-\frac{1}{2}i\Gamma \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \textbf{(A5)}$$

•Which has non-trivial solutions for

$$|\mathbf{H}-\lambda I| = 0$$

$$\Rightarrow \quad (M-\tfrac{1}{2}i\Gamma-\lambda)^2 - (M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)(M_{12}-\tfrac{1}{2}i\Gamma_{12}) = 0$$

with eigenvalues

$$\lambda = M-\tfrac{1}{2}i\Gamma \pm \sqrt{(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)(M_{12}-\tfrac{1}{2}i\Gamma_{12})}$$

•The eigenstates can be obtained by substituting back into **(A5)**

$$(M-\tfrac{1}{2}i\Gamma)x_1 + (M_{12}-\tfrac{1}{2}i\Gamma_{12}) = (M-\tfrac{1}{2}i\Gamma \pm \sqrt{(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)(M_{12}-\tfrac{1}{2}i\Gamma_{12})})x_1$$

$$\Rightarrow \qquad \frac{x_2}{x_1} = \pm\sqrt{\frac{M_{12}^*-\frac{1}{2}i\Gamma_{12}^*}{M_{12}-\frac{1}{2}i\Gamma_{12}}}$$

★ Define

$$\eta = \sqrt{\frac{M_{12}^*-\frac{1}{2}i\Gamma_{12}^*}{M_{12}-\frac{1}{2}i\Gamma_{12}}}$$

★ Hence the normalised eigenstates are

$$|K_\pm\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\begin{pmatrix} 1 \\ \pm\eta \end{pmatrix} = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle \pm \eta|\overline{K}^0\rangle)$$

★ Note, in the limit where M_{12}, Γ_{12} are real, the eigenstates correspond to the CP eigenstates **K_1** and **K_2**. Hence we can identify the general eigenstates as as the long and short lived neutral kaons:

$$\boxed{|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle + \eta|\overline{K}^0\rangle)} \qquad \boxed{|K_S\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle - \eta|\overline{K}^0\rangle)}$$

★ **Substituting these states back into (A2):**

$$\begin{aligned}
|\psi(t)\rangle &= a(t)|K^0\rangle + b(t)|\overline{K}^0\rangle \\
&= \sqrt{1+|\eta|^2}\left[\frac{a(t)}{2}(K_L+K_S) + \frac{b(t)}{2\eta}(K_L-K_S)\right] \\
&= \sqrt{1+|\eta|^2}\left[\left(\frac{a(t)}{2}+\frac{b(t)}{2\eta}\right)K_L + \left(\frac{a(t)}{2}-\frac{b(t)}{2\eta}\right)K_S\right] \\
&= \frac{\sqrt{1+|\eta|^2}}{2}\left[a_L(t)K_L + a_S(t)K_S\right]
\end{aligned}$$

with $$a_L(t) \equiv a(t) + \frac{b(t)}{\eta} \qquad a_S(t) \equiv a(t) - \frac{b(t)}{\eta}$$

★ **Now consider the time evolution of** $a_L(t)$

$$i\frac{\partial a_L}{\partial t} = i\frac{\partial a}{\partial t} + \frac{i}{\eta}\frac{\partial b}{\partial t}$$

★ **Which can be evaluated using (A4) for the time evolution of a(t) and b(t):**

$$\begin{aligned}
i\frac{\partial a_L}{\partial t} &= \left[(M-\tfrac{1}{2}i\Gamma_{12})a + (M_{12}-\tfrac{1}{2}i\Gamma_{12})b\right] + \frac{1}{\eta}\left[(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)a + (M-\tfrac{1}{2}i\Gamma)b\right] \\
&= (M-\tfrac{1}{2}i\Gamma)\left(a+\frac{b}{\eta}\right) + (M_{12}-\tfrac{1}{2}i\Gamma_{12})b + \frac{1}{\eta}(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)a \\
&= (M-\tfrac{1}{2}i\Gamma)a_L + (M_{12}-\tfrac{1}{2}i\Gamma_{12})b + \left(\sqrt{(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right)a \\
&= (M-\tfrac{1}{2}i\Gamma)a_L + \left(\sqrt{(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right)\left(a+\frac{b}{\eta}\right) \\
&= (M-\tfrac{1}{2}i\Gamma)a_L + \left(\sqrt{(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right)a_L \\
&= (m_L-\tfrac{1}{2}i\Gamma_L)a_L
\end{aligned}$$

★ **Hence:** $$\boxed{i\frac{\partial a_L}{\partial t} = (m_L-\tfrac{1}{2}i\Gamma_L)a_L}$$

with $$m_L = M + \Re\left\{\sqrt{(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right\}$$

and $$\Gamma_L = \Gamma - 2\Im\left\{\sqrt{(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)(M_{12}-\tfrac{1}{2}i\Gamma_{12})}\right\}$$

★ Following the same procedure obtain:

$$\boxed{i\frac{\partial a_S}{\partial t} = (m_S - \tfrac{1}{2}i\Gamma_S)a_S}$$

with $m_S = M - \Re\left\{\sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})}\right\}$

and $\Gamma_S = \Gamma + 2\Im\left\{\sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})}\right\}$

★ In matrix notation we have

$$\begin{pmatrix} M_L - \frac{1}{2}i\Gamma_L & 0 \\ 0 & M_S - \frac{1}{2}i\Gamma_S \end{pmatrix}\begin{pmatrix} a_L \\ a_S \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a_L \\ a_S \end{pmatrix}$$

★ Solving we obtain

$$a_L(t) \propto e^{-im_L t - \Gamma_L t/2} \qquad a_S(t) \propto e^{-im_S t - \Gamma_S t/2}$$

★ Hence in terms of the K_L and K_S basis the states propagate as independent particles with definite masses and lifetimes (the mass eigenstates). The time evolution of the neutral kaon system can be written

$$|\psi(t)\rangle = A_L e^{-im_L t - \Gamma_L t/2}|K_L\rangle + A_S e^{-im_S t - \Gamma_S t/2}|K_S\rangle$$

where A_L and A_S are constants

Appendix III: CP Violation : ππ decays

Non-examinable

★ Consider the development of the $K^0 - \overline{K}^0$ system now including CP violation

★ Repeat previous derivation using

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_1\rangle + \varepsilon|K_2\rangle] \qquad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle]$$

•Writing the CP eigenstates in terms of $K^0, \overline{K}^0$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K_0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

$$|K_S\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K_0\rangle - (1-\varepsilon)|\overline{K}^0\rangle\right]$$

•Inverting these expressions obtain

$$\boxed{|K^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}(|K_L\rangle + |K_S\rangle)} \qquad \boxed{|\overline{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1-\varepsilon}(|K_L\rangle - |K_S\rangle)}$$

•Hence a state that was produced as a K^0 evolves with time as:

$$|\psi(t)\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}(\theta_L(t)|K_L\rangle + \theta_S(t)|K_S\rangle)$$

where as before $\theta_S(t) = e^{-(im_S + \frac{\Gamma_S}{2})t}$ and $\theta_L(t) = e^{-(im_L + \frac{\Gamma_L}{2})t}$

•If we are considering the decay rate to ππ need to express the wave-function in terms of the CP eigenstates (remember we are neglecting CP violation in the decay)

$$\begin{aligned}|\psi(t)\rangle &= \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}[(|K_2\rangle+\varepsilon|K_1\rangle)\theta_L(t)+(|K_1\rangle+\varepsilon|K_2\rangle)\theta_S(t)] \\ &= \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}[(\theta_S+\varepsilon\theta_L)|K_1\rangle+(\theta_L+\varepsilon\theta_S)|K_2\rangle]\end{aligned}$$

CP Eigenstates

•Two pion decays occur with CP = +1 and therefore arise from decay of the CP = +1 kaon eigenstate, i.e. K_1

$$\Gamma(K^0_{t=0}\to\pi\pi)\propto|\langle K_1|\psi(t)\rangle|^2=\frac{1}{2}\left|\frac{1}{1+\varepsilon}\right|^2|\theta_S+\varepsilon\theta_L|^2$$

•Since $|\varepsilon|\ll 1$

$$\left|\frac{1}{1+\varepsilon}\right|^2=\frac{1}{(1+\varepsilon^*)(1+\varepsilon)}\approx\frac{1}{1+2\Re\{\varepsilon\}}\approx 1-2\Re\{\varepsilon\}$$

•Now evaluate the $|\theta_S+\varepsilon\theta_L|^2$ term again using

$$|z_1\pm z_2|^2=|z_1|^2+|z_2|^2\pm 2\Re(z_1z_2^*)$$

$$\begin{aligned}|\theta_S+\varepsilon\theta_L|^2 &= |e^{-im_St-\frac{\Gamma_S}{2}t}+\varepsilon e^{-im_Lt-\frac{\Gamma_L}{2}t}|^2 \\ &= e^{-\Gamma_St}+|\varepsilon|^2e^{-\Gamma_Lt}+2\Re\{e^{-im_St-\frac{\Gamma_S}{2}t}.\varepsilon^*e^{+im_Lt-\frac{\Gamma_L}{2}t}\}\end{aligned}$$

•Writing $\varepsilon=|\varepsilon|e^{i\phi}$

$$\begin{aligned}|\theta_S+\varepsilon\theta_L|^2 &= e^{-\Gamma_St}+|\varepsilon|^2e^{-\Gamma_Lt}+2|\varepsilon|e^{-(\Gamma_S+\Gamma_L)t/2}\Re\{e^{i(m_L-m_S)t-\phi}\} \\ &= e^{-\Gamma_St}+|\varepsilon|^2e^{-\Gamma_Lt}+2|\varepsilon|e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m.t-\phi)\end{aligned}$$

•Putting this together we obtain:

$$\boxed{\Gamma(K^0_{t=0}\to\pi\pi) = \frac{1}{2}(1-2\Re\{\varepsilon\})N_{\pi\pi}\left[e^{-\Gamma_St}+|\varepsilon|^2e^{-\Gamma_Lt}+2|\varepsilon|e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m.t-\phi)\right]}$$

Short lifetime component $K_S\to\pi\pi$

CP violating long lifetime component $K_L\to\pi\pi$

Interference term

•In exactly the same manner obtain for a beam which was produced as $\overline{K}^0$

$$\boxed{\Gamma(\overline{K}^0_{t=0}\to\pi\pi) = \frac{1}{2}(1+2\Re\{\varepsilon\})N_{\pi\pi}\left[e^{-\Gamma_St}+|\varepsilon|^2e^{-\Gamma_Lt}-2|\varepsilon|e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta m.t-\phi)\right]}$$

Interference term changes sign

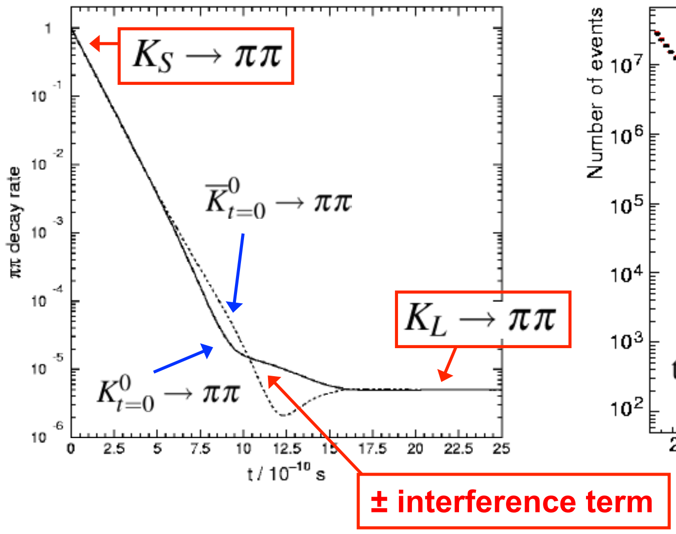
★ **At large proper times only the long lifetime component remains :**

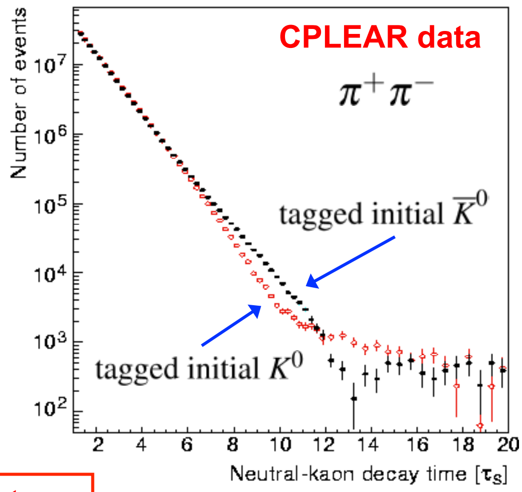
$$\Gamma(K^0_{t=0} \to \pi\pi) \quad \to \quad \frac{1}{2}(1 - 2\Re\{\varepsilon\})N_{\pi\pi}.|\varepsilon|^2 e^{-\Gamma_L t}$$

i.e. CP violating $K_L \longrightarrow \pi\pi$ decays

★ **Since CPLEAR can identify whether a K^0 or $\overline{K}^0$ was produced, able to measure $\Gamma(K^0_{t=0} \to \pi\pi)$ and $\Gamma(\overline{K}^0_{t=0} \to \pi\pi)$**

Prediction with CP violation





★ **The CPLEAR data shown previously can be used to measure $\varepsilon = |\varepsilon|e^{i\phi}$**

•**Define the asymmetry:**

$$A_{+-} = \frac{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) - \Gamma(K^0_{t=0} \to \pi\pi)}{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) + \Gamma(K^0_{t=0} \to \pi\pi)}$$

•**Using expressions on page 443**

$$A_{+-} = \frac{4\Re\{\varepsilon\}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - 4|\varepsilon|e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}{2\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - \underbrace{8\Re\{\varepsilon\}|\varepsilon|e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}}$$

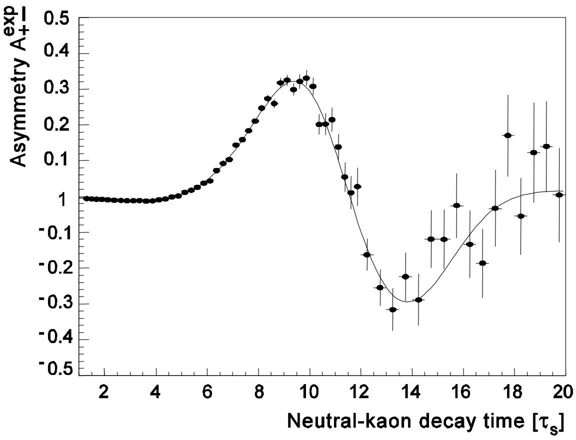
$\propto |\varepsilon|\Re\{\varepsilon\}$ **i.e. two small quantities and can safely be neglected**

$$A_{+-} \approx \frac{2\Re\{\varepsilon\}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - 2|\varepsilon|e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon|e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon|e^{(\Gamma_S-\Gamma_L)t/2}\cos(\Delta m.t - \phi)}{1 + |\varepsilon|^2 e^{(\Gamma_S-\Gamma_L)t}}$$

A.Apostolakis et al., Eur. Phys. J. C18 (2000) 41



Best fit to the data:

$$|\varepsilon| = (2.264 \pm 0.035) \times 10^{-3}$$
$$\phi = (43.19 \pm 0.73)^\circ$$

Appendix IV: CP Violation via Mixing

Non-examinable

★ A full description of the SM origin of CP violation in the kaon system is beyond the level of this course, nevertheless, the relation to the box diagrams is illustrated below

★ The K-long and K-short wave-functions depend on η

$$|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle + \eta|\overline{K}^0\rangle) \qquad |K_S\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle - \eta|\overline{K}^0\rangle)$$

with $$\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

★ If $M_{12}^* = M_{12};\quad \Gamma_{12}^* = \Gamma_{12}$ then the K-long and K-short correspond to the CP eigenstates K_1 and K_2

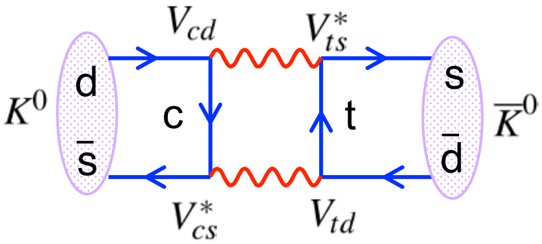
•CP violation is therefore associated with imaginary off-diagonal mass and decay elements for the neutral kaon system

•Experimentally, CP violation is small and $\eta \approx 1$

•Define: $\varepsilon = \frac{1-\eta}{1+\eta} \quad \Rightarrow \quad \eta = \frac{1-\varepsilon}{1+\varepsilon}$

•Consider the mixing term M_{12} which arises from the sum over all possible intermediate states in the mixing box diagrams

e.g.



$$M_{12} = A_{ct} V_{cd} V_{cs}^* V_{ts}^* V_{td} + \ldots$$

•Therefore it can be seen that, in the Standard Model, CP violation is associated with the imaginary components of the CKM matrix

•It can be shown that mixing leads to CP violation with

$$|\varepsilon| \propto \Im\{M_{12}\}$$

•The differences in masses of the mass eigenstates can be shown to be:

$$\Delta m_K = m_{K_L} - m_{K_S} \approx \sum_{q,q'} \frac{G_F^2}{3\pi^2} f_K^2 m_K |V_{qd} V_{qs}^* V_{q'd} V_{q's}^*| m_q m_{q'}$$

where q and q' are the quarks in the loops and f_K is a constant

•In terms of the small parameter ε

$$|K_L\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

$$|K_S\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1-\varepsilon)|K^0\rangle + (1+\varepsilon)|\overline{K}^0\rangle\right]$$

★ If epsilon is non-zero we have CP violation in the neutral kaon system

Writing $\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}} = \sqrt{\frac{z^*}{z}}$ and $z = ae^{i\phi}$

gives $\eta = e^{-i\phi}$

★ From which we can find an expression for ε

$$\varepsilon.\varepsilon^* = \frac{1-e^{-i\phi}}{1+e^{-i\phi}} \cdot \frac{1-e^{+i\phi}}{1+e^{i\phi}} = \frac{2-\cos\phi}{2+\cos\phi} = \tan^2\frac{\phi}{2}$$

$$|\varepsilon| = |\tan\tfrac{\phi}{2}|$$

★ Experimentally we know ε is small, hence ϕ is small

$$|\varepsilon| \approx \frac{1}{2}\phi = \frac{1}{2}\arg z \approx \frac{1}{2}\frac{\Im\{M_{12} - \frac{1}{2}i\Gamma_{12}\}}{|M_{12} - \frac{1}{2}i\Gamma_{12}|}$$

Appendix V: Time Reversal Violation

•Previously, equations (4) and (5), obtained expressions for strangeness oscillations in the absence of CP violation, e.g.

$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

•This analysis can be extended to include the effects of CP violation to give the following rates (see question 24):

$$\Gamma(K^0_{t=0} \to K^0) \propto \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$
$$\Gamma(\overline{K}^0_{t=0} \to \overline{K}^0) \propto \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$
$$\Gamma(\overline{K}^0_{t=0} \to K^0) \propto \tfrac{1}{4}\boxed{(1+4\Re\{\varepsilon\})}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$
$$\Gamma(K^0_{t=0} \to \overline{K}^0) \propto \tfrac{1}{4}\boxed{(1-4\Re\{\varepsilon\})}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

★ Including the effects of CP violation find that

$$\boxed{\Gamma(\overline{K}^0_{t=0} \to K^0) \neq \Gamma(K^0_{t=0} \to \overline{K}^0)}$$

Violation of time reversal symmetry !

★ No surprise, as CPT is conserved, CP violation implies T violation