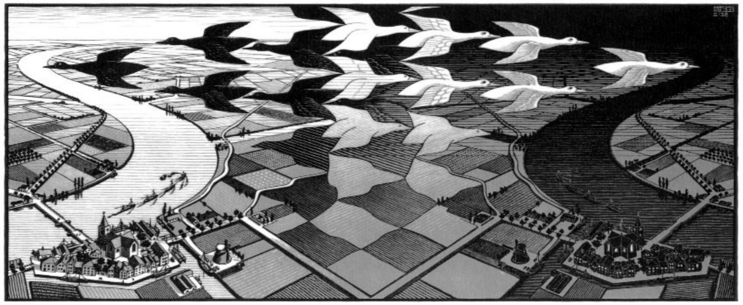

Kaons story
CP-Violation in kaon decays

Agnieszka Obłąkowska-Mucha, Tomasz Szumlak

AGH-UST Krakow

$\theta - \tau$ puzzle

- In 1949 C.F. Powell discovered in cosmic rays:
  - π meson,
  - a meson that disintegrated into three pions (named τ meson),
  - another particle (θ) that decays into two pions had been known that time.

$$\theta^0 \to \pi^0 + \pi^0$$

$$\theta^0 \to \pi^+ + \pi^-$$

$$\tau^0 \to \pi^0 + \pi^0 + \pi^0$$

$$\tau^0 \to \pi^+ + \pi^- + \pi^0$$

- θ and τ particles turned out to be indistinguishable other than their mode of decay. Their masses and lifetimes were identical, within the experimental uncertainties. **Were they in fact the same particle?**
- If the CP symmetry is valid, θ and τ cannot be the same particle.
- **First doubts arose... that *P* parity is not conserved in weak interaction** (confirmed by Wu experiment).

$\theta - \tau$ puzzle

Behavior of Neutral Particles under Charge Conjugation

M. Gell-Mann,* *Department of Physics, Columbia University, New York, New York*

and

A. Pais, *Institute for Advanced Study, Princeton, New Jersey*

(Received November 1, 1954)

Some properties are discussed of the θ^0, a heavy boson that is known to decay by the process $\theta^0 \to \pi^+ + \pi^-$. According to certain schemes proposed for the interpretation of hyperons and K particles, the θ^0 possesses an antiparticle $\bar{\theta}^0$ distinct from itself. Some theoretical implications of this situation are discussed with special reference to charge conjugation invariance. The application of such invariance in familiar instances is surveyed in Sec. I. It is then shown in Sec. II that, within the framework of the tentative schemes under consideration, the θ^0 must be considered as a "particle mixture" exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that no more than half of all θ^0's undergo the familiar decay into two pions. Some experimental consequences of this picture are mentioned.

$\theta - \tau$ puzzle

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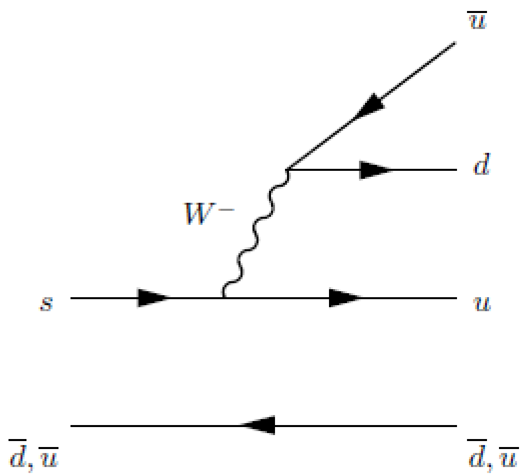
Some properties are discussed of the K^0 a heavy boson that is known to decay by the process $K^0 \to \pi^+ + \pi^-$. According to certain schemes proposed for the interpretation of hyperons and K particles, the K^0 possesses an antiparticle $\bar{K}^0$ distinct from itself. Some theoretical implications of this situation are discussed with special reference to charge conjugation invariance. The application of such invariance in familiar instances is surveyed in Sec. I. It is then shown in Sec. II that, within the framework of the tentative schemes under consideration, the K^0 must be considered as a "particle mixture" exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that no more than half of all K^0's undergo the familiar decay into two pions. Some experimental consequences of this picture are mentioned.

- How can we distinguish K^0 from $\bar{K}^0$?

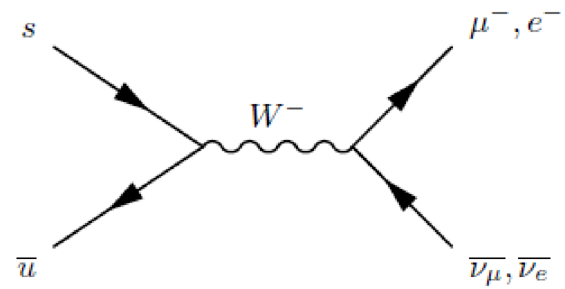
Oscillations of Neutral Mesons

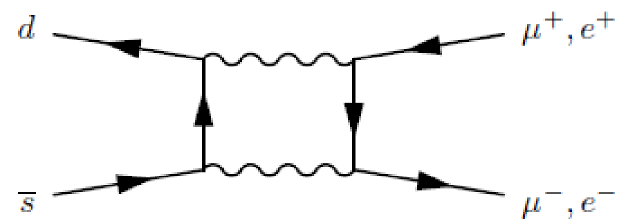
- The real questions here:
  - How (θ^0, τ^0) are related to $(K^0, \bar{K}^0)$?
  - Are K^0 different than $\bar{K}^0$?
- This is not trivial…
- Using purely hadronic and leptonic decays, we cannot distinguish them

Isospin	-1	+1
+1	$\overline{K^0}$ $(s\bar{d})$	K^+ $(\bar{s}u)$
-1	K^- $(s\bar{u})$	K^0 $(\bar{s}d)$
		"Strangeness"



$$
\begin{aligned}
K^+ &\rightarrow \pi^+\pi^0,\ \pi^+\pi^-\pi^+,\ \pi^+\pi^0\pi^0 \\
K^- &\rightarrow \pi^-\pi^0,\ \pi^-\pi^+\pi^-,\ \pi^-\pi^0\pi^0 \\
K^0 &\rightarrow \pi^0\pi^0,\ \pi^0\pi^0\pi^0,\ \pi^+\pi^-,\ \pi^+\pi^-\pi^0 \\
\overline{K^0} &\rightarrow \pi^0\pi^0,\ \pi^0\pi^0\pi^0,\ \pi^+\pi^-,\ \pi^+\pi^-\pi^0
\end{aligned}
$$

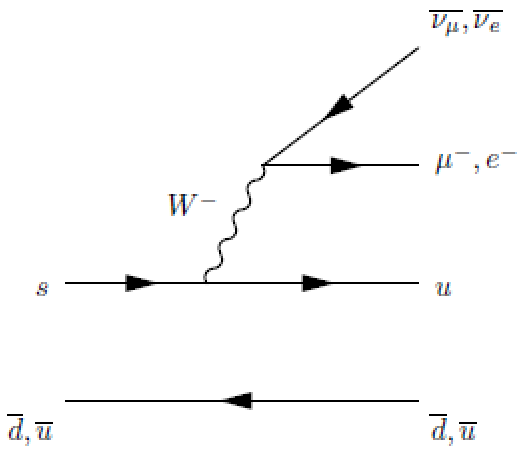


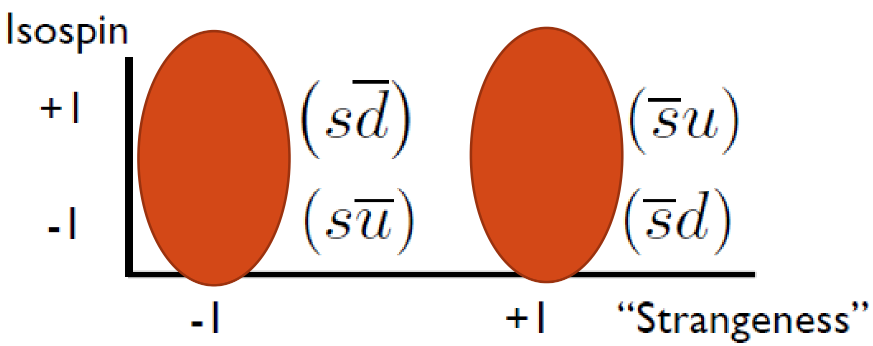


$$
\begin{aligned}
K^+ &\rightarrow \mu^+\nu_\mu,\ e^+\nu_e \\
K^- &\rightarrow \mu^-\overline{\nu_\mu},\ e^-\overline{\nu_e} \\
K^0 &\rightarrow \mu^-\mu^+,\ e^-e^+ \\
\overline{K^0} &\rightarrow \mu^+\mu^-,\ e^+e^-
\end{aligned}
$$

Oscillations of Neutral Mesons

- The real questions here:
  - How (θ^0, τ^0) are related to $(K^0, \overline{K}^0)$?
  - Are K^0 different than $\overline{K}^0$?
- This is not trivial…
- Now, semileptonic…





$$
\begin{aligned}
K^+ &\rightarrow \pi^0\mu^+\nu_\mu,\ \pi^0 e^+\nu_e \\
K^- &\rightarrow \pi^0\mu^-\overline{\nu_\mu},\ \pi^0 e^-\overline{\nu_e} \\
K^0 &\rightarrow \pi^-\mu^+\nu_\mu,\ \pi^- e^+\nu_e \\
\overline{K^0} &\rightarrow \pi^+\mu^-\overline{\nu_\mu},\ \pi^+ e^-\overline{\nu_e}
\end{aligned}
$$

Oscillations of Neutral Mesons

- These neutral kaons are produced in the **strong** interactions with well **defined strangeness**, i.e., as eigenstates of the $\mathcal{S}$ operator

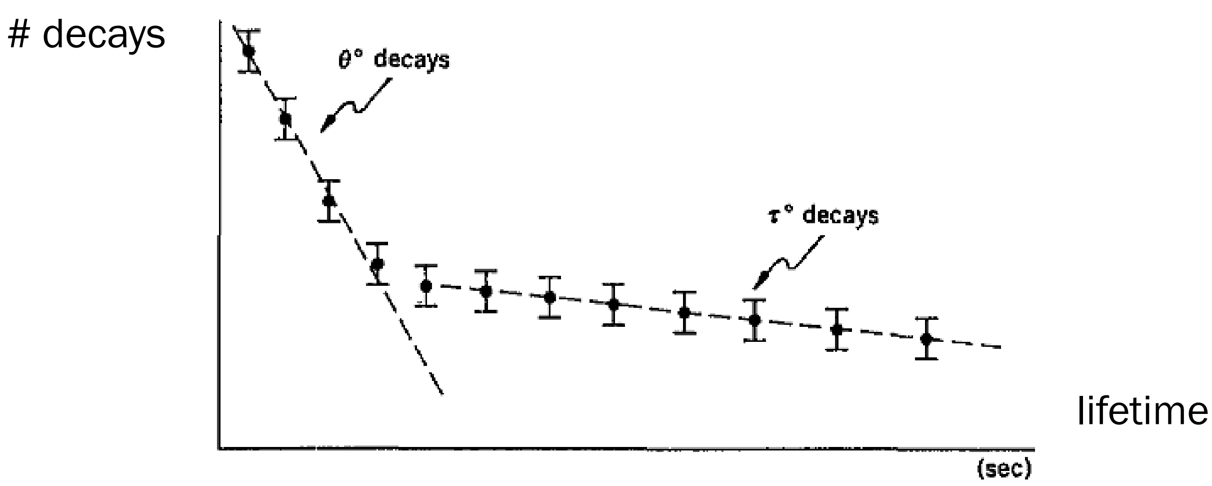
$$\mathcal{S}|K^0\rangle = +1|K^0\rangle, \mathcal{S}|\bar{K}^0\rangle = -1|\bar{K}^0\rangle$$

$$K^- + p = \bar{K}^0 + n$$

$$K^+ + n = K^0 + p$$

$$\pi^- + p = \Lambda^0 + K^0$$

- Thus, K^0 is an antiparticle of $\bar{K}^0$ and they **can be tell apart** by the value of their strangeness!
- After production by the strong forces the kaons are unstable and decay – we can measure their lifetimes. Since they are antiparticles for each other we expect (the $\mathcal{CPT}$ theorem) that their **masses** and **lifetimes** are the same!
- **Instead a remarkable result**



Oscillations of Neutral Mesons

- Instead of **well defined** (single!) lifetime, as expected from a unique eigenstate of free-particle Hamiltonian, the **data** indicate **two distinct** lifetimes related to both K^0 and $\overline{K}^0$
- K^0 and $\overline{K}^0$ must be **superposition** of two distinct states with different lifetimes
- We call them K_1^0 (two pion channels) and K_2^0 (three pion channels)
- The results found for K^0 and $\overline{K}^0$ are then consistent in the sense that the lifetimes found for both their **components** K_1^0 and K_2^0 are **the same**!

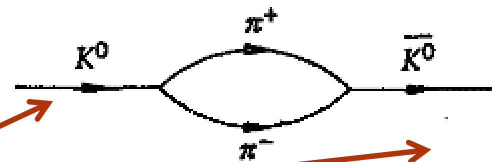
$$\tau_1 \approx 0.9 \times 10^{-10}\ s$$

$$\tau_2 \approx 5.0 \times 10^{-8}\ s$$

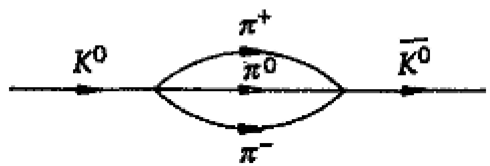
- One more thing, since K^0 and $\overline{K}^0$ share the same decay channels we say that they can **mix with each** other via higher order weak interactions
- Although they are produced as unique states (different S) they propagate in time as a mixture of states (the same decay channels)

Oscillations of Neutral Mesons

- To be more precise: K^0 and $\overline{K}^0$ are produced as orthogonal states
- This orthogonality is then broken by the weak interactions and the transition $K^0 \leftrightarrow \overline{K}^0$ is possible – the weak interaction do not conserve strangeness



Kaons can mix!
Fantastic property!



- K^0 and $\overline{K}^0$ are the eigenstates of the **strong** hamiltonian but **cannot be** the eigenstates of the **weak** interactions!

$$\langle K^0 | \overline{K}^0 \rangle = 0 \;\rightarrow \langle K^0 | H_{Strong} | \overline{K}^0 \rangle = 0$$

$$H_{Strong} | K^0 \rangle = m_{K^0} | K^0 \rangle \qquad H_{Strong} | \overline{K}^0 \rangle = m_{\overline{K}^0} | \overline{K}^0 \rangle$$

$$m_{K^0} = m_{\overline{K}^0} \approx 498\ MeV$$

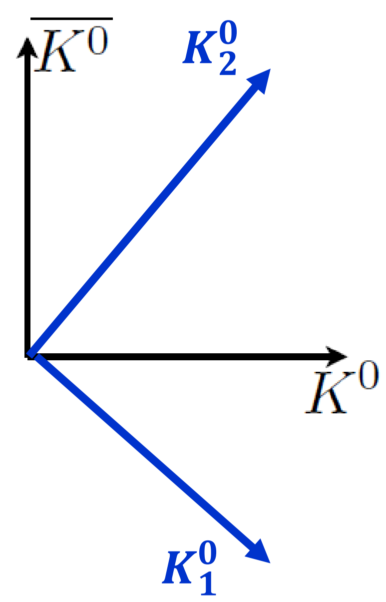
Oscillations of Neutral Mesons

- For the weak interactions we have then

$$\boxed{\langle K^0 | H_{Weak} | \bar{K}^0 \rangle \neq 0}$$

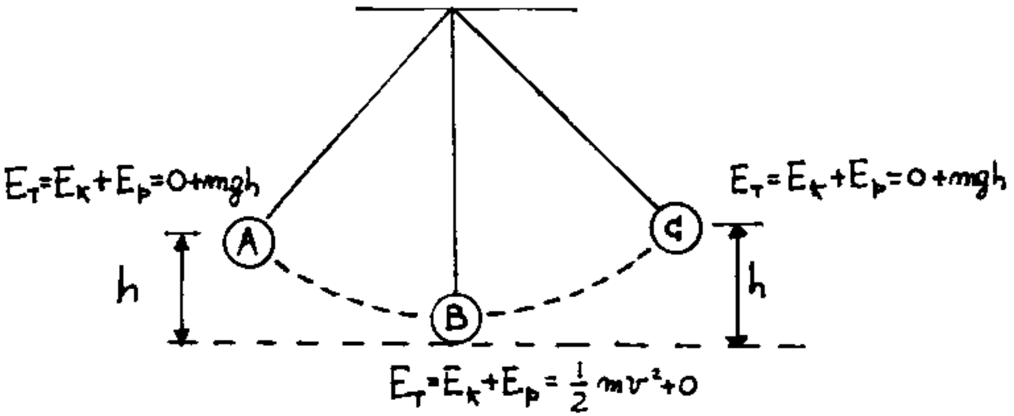
- Kaons decay in weak processes, given that K_1^0 and K_2^0 have unique lifetimes we can treat them as **eigenstates** of H_{Weak}

- Now quantum physics starts twist our brains… Since we used the picture where K^0 and $\bar{K}^0$ are a **mixture** of K_1^0 and K_2^0 to explain the weird lifetime data now we can say that K_1^0 and K_2^0 are **mixture** of K^0 and $\bar{K}^0$ - this makes description of the **mass** states much nicer!



Introduction, i.e., the Big Picture

- The way to attack this problem in HEP is to understand
  - What the Universe is built of – „**matter particles**”
  - How these matter particles interact – **forces** (also particles…)
- The most successful recipe is the **Standard Model** which is based on principle of **gauge invariance** = symmetry
- In other words – **forces** are **consequence** of various **symmetries**, in order to study them we need to understand their invariance principles
- Let check this out – familiar example – **energy conservation**



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- Let check this out – familiar example – **energy conservation**

Macroscopic (classic) **gravity force** is **invariant** under **time translation**

Symmetry w.r.t. time translations = **conservation** of Energy

Introduction, i.e., the Big Picture

- The way to attack this problem in HEP is to understand
  - What the Universe is built of – „**matter particles**”
  - How these matter particles interact – **forces** (also particles…)
- The most successful recipe is the **Standard Model** which is based on principle of **gauge invariance** = symmetry
- In other words – **forces** are **consequence** of various **symmetries**, in order to study them we need to understand their invariance principles
- Let check this out – and not so familiar example…

Invariance w.r.t. **arbitrary change** of a wave function **phase** – **electric charge** conservation (gauge transformation)

Absolute phase of a quantum state cannot be measured

Introduction, i.e., the Big Picture

- There is more… Discrete symmetries! $\mathcal{C}, \mathcal{P}, \mathcal{T}$
  - $\mathcal{C}$ – particle anti-particle conjugation (change sign of all additive quantum numbers…, eh, not quite classical…)
  - $\mathcal{P}$ – mirror symmetry (reflection in a plane mirror and a rotation by 180^o)
  - $\mathcal{T}$ – time reversal (formal reversing the sign of the time axis)
- Known and used in classical physics for quite some time, regarded as just something curies (quantum physics made them great!)
- Classical physics treats time and charge conjugations as trivial
- More interesting stuff going on with the parity

$$\mathcal{P}\vec{r} \rightarrow -\vec{r}$$

Polar vector

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt}, \vec{p} = m\vec{v}, \vec{F} = \frac{d\vec{p}}{dt}}$$

Introduction, i.e., the Big Picture

$$\vec{F}_L = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \to \mathcal{P}\vec{F}_L = -\vec{F}_L \to \mathcal{P}\vec{B} = \vec{B}$$

Axial vector

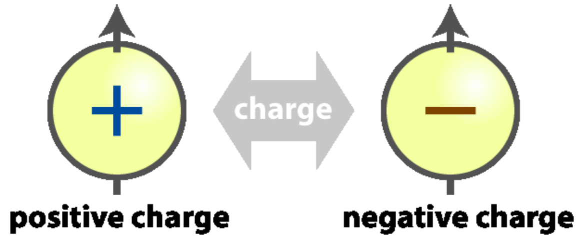
- Already within the framework of the classical physics we can have four classes of quantities with different behavior under parity transformation
  - **Scalars** (m)
  - **(Polar) Vectors** $(\vec{p},\ \vec{F})$
  - **Pseudo-scalars** (e.g., $\vec{E} \cdot \vec{B}$)
  - **(Axial Vectors) Pseudo-vectors** $(\vec{B}, \vec{L})$

- Nice, but let's see what the quantum theory does for us…

Introduction, i.e., the Big Picture

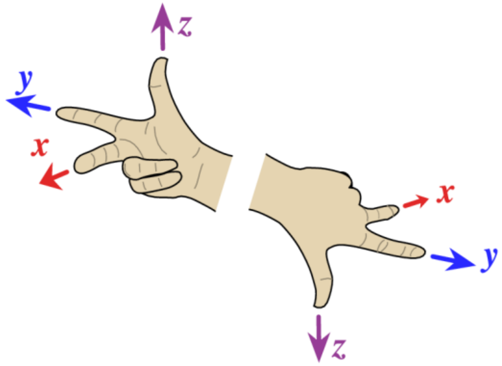
- $\mathcal{C}$ – formally changes a field ϕ into a related one $\phi^{\dagger}$, the latter one has just all its additive quantum numbers with opposite signs
  - Charge
  - Lepton number
  - Barion number
  - …

- We know based on experimental work that the invariance under $\mathcal{C}$ transformation always holds for the strong and e-m interactions
- Cannot distinguish between matter and anti-matter using any observable related to strong or e-m forces!



Introduction, i.e., the Big Picture

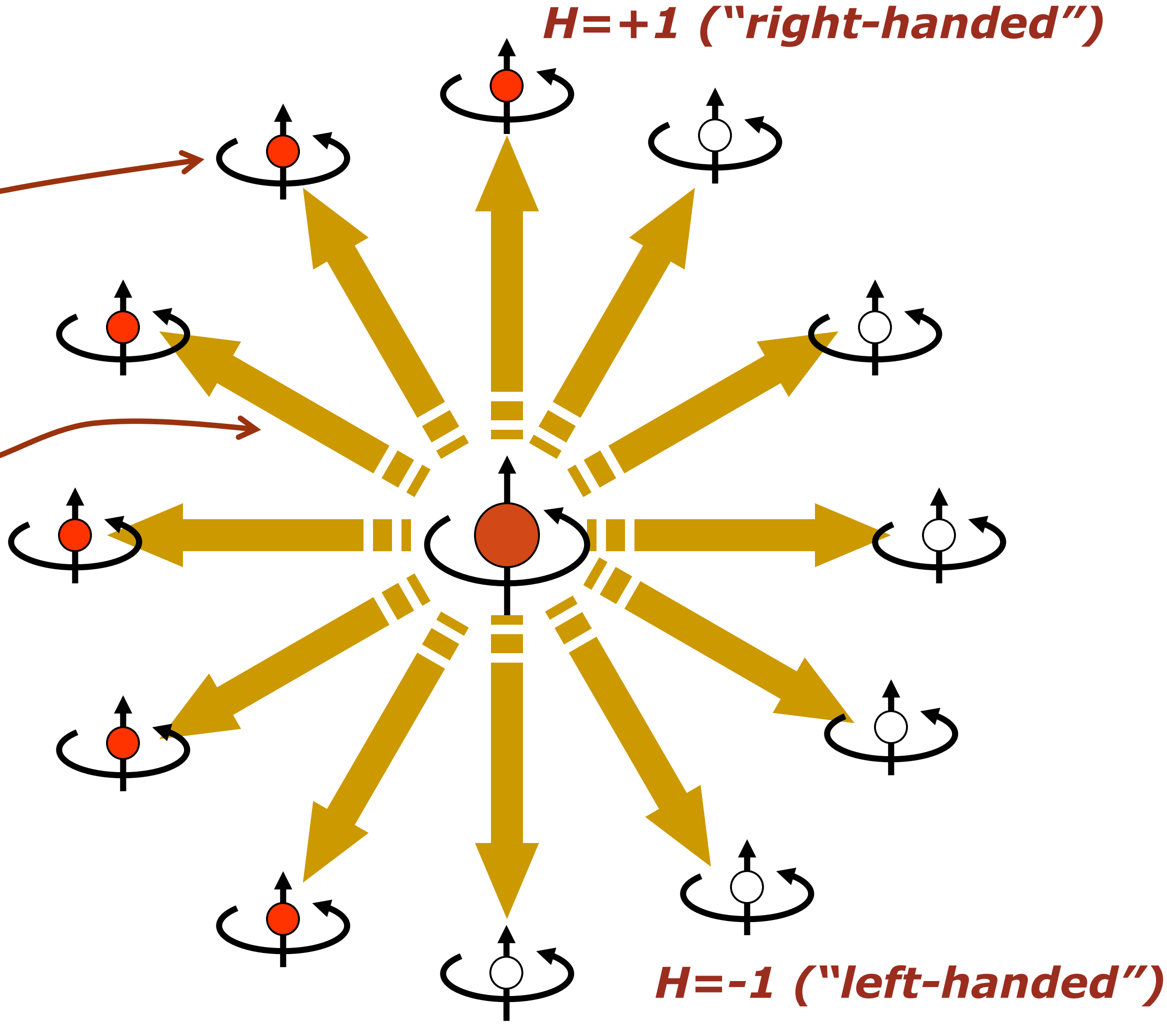
- $\mathcal{P}$ – parity invariance regarded as „common sense”, why physics would distinguish between the real and mirror worlds? No way…
- But, we got so called $\theta - \tau$ puzzle (see next slides)
  - To deal with it, the theorists realised that the weak interactions must be described by quantities that are mixture of vectors and pseudo-vectors (V-A theory)!
  - That was a huge step forward in getting to the SM
  - Now this may lead to quantities that will behave as pseudo-scalars under parity transformation, thus…
  - the difference between the real and mirror world!



Introduction, i.e., the Big Picture

- So, what such **pseudo-scalar observable** would look like? Meet the fantastic **helicity**! (Well, meet it the second time, see the last lecture…)

$$H = \frac{\vec{s} \cdot \vec{p}}{|\vec{s} \cdot \vec{p}|}$$



A Small Detour – Parity Operator

- $\mathcal{P}$ – operator and its eigenstates
  - Two successive parity transformation leave a vector unchanged

$$\mathcal{P}\vec{r} \to -\vec{r}, \mathcal{P}(-\vec{r}) \to \vec{r}$$

  - this gives us:

$$\mathcal{P}\ \mathcal{P}|\alpha\rangle = \mathcal{P}^2|\alpha\rangle = +1|\alpha\rangle$$

  - this is known fact – parity operator eigenvalues can only be ± 1
- So, for any **parity invariant** Hamiltonian the following is true:

$$\boxed{\left[\mathcal{P}, \widehat{H}\right] = 0}$$

- If both operators **commute** the eigenstates of the Hamiltonian are also eigenstates of parity operator with eigenvalues of either $+1$ or -1
- Since wave function transforms under parity as follow: $\mathcal{P}\alpha(\vec{r}) = \alpha(-\vec{r})$, this implies that any stationary eigenstates of parity invariant Hamiltonian **have definite parity**!
- We call them **odd** and **even** states

Oscillations of Neutral Mesons

- We saw that the weak interactions **maximally violate** charge and space parities
- Also, there was a hint that the **combined symmetry** $\mathcal{CP}$ may be **exact** one
- Invariance under $\mathcal{CP}$ implies **matter – anti-matter symmetry**
- Ok, wait a moment… we know this is **not true**! Just look out in the night! The Universe is **dominated** by matter…
- So, $\mathcal{CP}$ cannot be the exact symmetry of the Universe! Are there any hints regarding **breaking the combined symmetry**?
- Let’s have a look at $\theta - \tau$ puzzle again…

$$\theta^0 \to \pi^0 + \pi^0$$

$$\theta^0 \to \pi^+ + \pi^-$$

$$\tau^0 \to \pi^0 + \pi^0 + \pi^0$$

$$\tau^0 \to \pi^+ + \pi^- + \pi^0$$

$$K^- + p = \bar{K}^0 + n$$

$$K^+ + n = K^0 + p$$

$$\pi^- + p = \Lambda^0 + K^0$$

What is the connection with neutral **Kaons**?

Oscillations of Neutral Mesons

- For the weak interactions we have then

$$\boxed{\langle K^0 | H_{Weak} | \overline{K}^0 \rangle \neq 0}$$

- Kaons decay in weak processes, given that K_1^0 and K_2^0 have unique lifetimes we can treat them as **eigenstates** of H_{Weak}
- Now quantum physics starts twist our brains… Since we used the picture where K^0 and $\overline{K}^0$ are a **mixture** of K_1^0 and K_2^0 to explain the weird lifetime data now we can say that K_1^0 and K_2^0 are **mixture** of K^0 and $\overline{K}^0$ - this makes description of the **mass** states much nicer!
- Just follow to the next slide…

Neutral Mesons and CP

- Let's start with the assumption that $\mathcal{CP}$ is a good symmetry of the weak interactions
- Kaons are pseudo-scalars, thus, have odd intrinsic parities

$$\mathcal{CP}|K^0\rangle = -\mathcal{C}|K^0\rangle = -|\bar{K}^0\rangle$$

$$\mathcal{CP}|\bar{K}^0\rangle = -\mathcal{C}|\bar{K}^0\rangle = -|K^0\rangle$$

- Can use appropriate linear orthonormal combinations that are eigenstates of $\mathcal{CP}$ operator

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$$\mathcal{CP}|K_1^0\rangle = \frac{1}{\sqrt{2}}(\mathcal{CP}|K^0\rangle - \mathcal{CP}|\bar{K}^0\rangle) = \frac{1}{\sqrt{2}}(-|\bar{K}^0\rangle + |K^0\rangle) = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) = |K_1^0\rangle$$

$$\mathcal{CP}|K_2^0\rangle = \cdots = -|K_2^0\rangle$$

Neutral Mesons and CP

- Now, K_1^0 and K_2^0 can be regarded as **eigenstates of $\mathcal{CP}$** with even and odd eigenvalues respectively
- One extraordinary thing – **cannot** define **unique strangeness** of these states!
- Now can identify them as

$$\boxed{\theta^0 \equiv K_1^0 \rightarrow \pi^0 + \pi^0}$$

$$\boxed{\tau^0 \equiv K_2^0 \rightarrow \pi^0 + \pi^0 + \pi^0}$$

- Since the phase space (density of states) for two body decay is much larger than for three body one
- The rate of decay for K_1^0 should be much larger than for K_2^0
- Or in other words - K_1^0 lifetime should be much shorter than for K_2^0
- This is what the experiment showed us. Great!

Flavour (strangeness) oscillation

- Strong interaction gives us kaons with **definite** strangeness, we write down the following:

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + |K_2^0\rangle\right)$$

$$|\overline{K}^0\rangle = -\frac{1}{\sqrt{2}}\left(|K_1^0\rangle - |K_2^0\rangle\right)$$

- Kaons are **produced** as eigenstates of **strong Hamiltonian** (mixture of weak Hamiltonian states) but **propagate** through time as eigenstates of **weak one**
- In time both components of strong states **decay away** and after a sufficient amount of time we are going to have only $|K_2^0\rangle$ component
- However, since $|K_2^0\rangle$ is a mixture of $|K^0\rangle$ and $|\overline{K}^0\rangle$ states, even starting from pure $|K^0\rangle$ (or $|\overline{K}^0\rangle$) state we end up with a mixture of states of different strangeness
- This phenomenon is called **flavour oscillation**

Flavour (strangeness) oscillation

- This effect can be measured! Just need to put the anti-kaons in some medium and observe them interacting strongly with it (because strong interaction preserve strangeness!)

$$\bar{K}^0 + p \to \Sigma^+ + \pi^+ + \pi^-$$

$$\bar{K}^0 + p \to \Lambda^0 + \pi^+ + \pi^0$$

$$K^0 + p \not\to \Sigma^+ + \pi^+ + \pi^-$$

$$K^0 + p \not\to \Lambda^0 + \pi^+ + \pi^0$$

- Detecting hiperons is a proof of $\bar{K}^0$ presence!
- Similar oscillation effects for beauty and charm mesons!

Violation of CP symmetry

- Remember – K_1^0 and K_2^0 are eigenstates of CP operator

$$\mathcal{CP}|K_1^0\rangle = \frac{1}{\sqrt{2}}(\mathcal{CP}|K^0\rangle - \mathcal{CP}|\overline{K}^0\rangle) = \frac{1}{\sqrt{2}}(-|\overline{K}^0\rangle + |K^0\rangle) = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) = |K_1^0\rangle$$

- In other words, if combined parity is conserved, processes such these below should never happen!

$$K_2^0 \not\to \pi^0 + \pi^0$$

$$K_2^0 \not\to \pi^+ + \pi^-$$

- We should not be surprised by the fact that they **indeed happen**! An experiment has been performed to study the behaviour of the long-lived component of K^0, which **found them**!
- So, we are for another redefinition of what the kaons really are…
- Because we see clearly that CP **is broken**, thus, we must accept that neutral kaons **are not composed** out of K_1^0 and K_2^0
- The new states are called $\boldsymbol{K_S^0}$ and $\boldsymbol{K_L^0}$ instead…

Violation of CP symmetry

- This may come as yet another surprise, but the effect is **very weak**, the fractional branching ratios measured are of order of 0.1%

$$\frac{K_L^0 \to \pi^+ + \pi^-}{K_L^0 \to anything} \approx 2.0 \times 10^{-3} \qquad \frac{K_L^0 \to \pi^0 + \pi^0}{K_L^0 \to anything} \approx 9.0 \times 10^{-4}$$

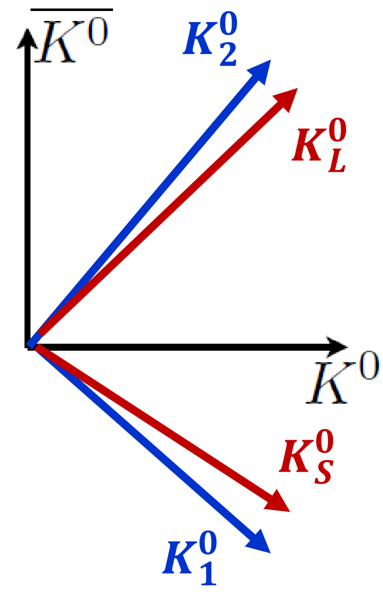
- Taking into account life-times of both K_S^0 and K_L^0 one can show that

$$\frac{\Gamma(K_L^0 \to 2\pi)}{\Gamma(K_S^0 \to 2\pi)} \approx 10^{-6}$$

- Ok – a small resume…
  - $\boldsymbol{CP}$ is indeed **violated**
  - The effect is **tiny** (not so tiny for beauty decays though…)
  - Matter and anti-matter are not **symmetrical**
  - CP, apart from small number of weak processes involving neutral mesons, is conserved

Kaons revisited

- What we did was an attempt to describe time evolution of kaons which are produced as **strong** Hamiltonian **e-states** that, in turn, decay as weak e-states: $(K^0, \overline{K}^0) \rightarrow (K_1^0, K_2^0)$
- This fails because K_1^0 and K_2^0 **are e-states of** $\boldsymbol{CP}$, so we need new particles, namely K_S^0 and K_L^0 that have the necessary behavior of K_1^0 and K_2^0 (i.e., long and short life-time) but are not CP e-states
- One remark – since the violation effect is small – this would be a hint that these new states are almost identical to K_1^0 and K_2^0



Kaons revisited

- One remark – since the violation effect is small – this would be a hint that these new states are almost identical to K_1^0 and K_2^0

$$|K_S^0\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\big((1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\big) =$$
$$= \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\big((|K^0\rangle - |\bar{K}^0\rangle) + \epsilon(|K^0\rangle + |\bar{K}^0\rangle)\big) =$$
$$= \frac{1}{\sqrt{(1+|\epsilon|^2)}}\big(|K_1^0\rangle + \epsilon|K_2^0\rangle\big)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\big((1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle\big) =$$
$$= \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\big((|K^0\rangle + |\bar{K}^0\rangle) + \epsilon(|K^0\rangle - |\bar{K}^0\rangle)\big) =$$
$$= \frac{1}{\sqrt{(1+|\epsilon|^2)}}\big(|K_2^0\rangle + \epsilon|K_1^0\rangle\big)$$

Kaons Revisited

- How should we thing about what is going on..., so,...
- The transitions $\langle 2\pi|H_w|K_L^0\rangle$ violate CP invariance. This can happen:
  - because the e-states of the weak Hamiltonian, K_S^0 and K_L^0, are not e-states of the CP operator
  - we say, that the physical states are **mixtures** of $\boldsymbol{CP}$**-even** and $\boldsymbol{CP}$**-odd** components
  - In other words – we observe small violation of CP in $K_L^0 \to 2\pi$ decays, because of small admixture of K_1^0
  - this type of violation is called **indirect**, and implies, that the Hamiltonian itself is even under CP symmetry
  - again... to add confusion, it turns out that the **direct** violation is also possible for kaons (i.e., violation induced via the weak Hamiltonian) $\langle 2\pi|H_w|K_2^0\rangle \neq 0$
  - But that is another story...

Measure of CP-violation

- How can we express the degree of CP-violation?

$$|K_S^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_1^0\rangle + \epsilon|K_2^0\rangle\right) \qquad |K_L^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_2^0\rangle + \epsilon|K_1^0\rangle\right)$$

ϵ represents deviation of K_S^0 and K_L^0 from true CP e-states (in general this is complex number!)

$$\mathcal{CP}|K_S^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(\mathcal{CP}|K_1^0\rangle + \epsilon\mathcal{CP}|K_2^0\rangle\right) = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_1^0\rangle - \epsilon|K_2^0\rangle\right) \neq |K_S^0\rangle$$

$$\mathcal{CP}|K_L^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(\mathcal{CP}|K_2^0\rangle + \epsilon\mathcal{CP}|K_1^0\rangle\right) = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(-|K_1^0\rangle + \epsilon|K_2^0\rangle\right) \neq -|K_L^0\rangle$$

$$\langle K_L^0|K_S^0\rangle = \frac{1}{1+|\epsilon|^2}\left(\langle K_2^0| + \epsilon^*\langle K_1^0|\right)\left(|K_1^0\rangle + \epsilon|K_2^0\rangle\right) =$$

$$\frac{1}{1+|\epsilon|^2}\left(\epsilon\langle K_2^0|K_2^0\rangle + \epsilon^*\langle K_1^0|K_1^0\rangle\right) = \frac{\epsilon+\epsilon^*}{1+|\epsilon|^2} = \frac{2Re(\epsilon)}{1+|\epsilon|^2} = \langle K_S^0|K_L^0\rangle$$

K_S^0 and K_L^0 are not orthogonal states!

Measure of CP-violation

- Lack of orthogonality of K_S^0 and K_L^0 is expected – both of them **share** the same decay channels
- This effect is at the same time a **measure** of CP-violation via $\boldsymbol{\epsilon}$
- In this picture the symmetry violation is a consequence of small admixture of K_1^0 state into the K_L^0, so, we observe its decays to 2π final state because the K_1^0 can decay into it – once again this is **indirect process**
- These kind of processes are referred to as $\boldsymbol{\Delta S = 2, \Delta I = \frac{1}{2}}$ transitions
- Much smaller direct contribution to CP-violation is a consequence of the **weak Hamiltonian having a $\boldsymbol{CP}$-violating term** (it does not commute with the CP operator)
- These kind of processes proceed via $\boldsymbol{\Delta S = 1, \Delta I = \frac{3}{2}}$ transitions and are called **penguin** (or loop) decays

Time Evolution of the Kaon System

- A phenomenological „**effective**" theoretical framework has been introduced to describe what is going on with kaons produced in strong interactions
- It is based on perturbation theory and describe the behavior of such system in terms of an effective Hamiltonian
- We start with describing kaons in the absence of weak interactions
  - In this case K^0 and $\bar{K}^0$ are distinct e-states of the strong Hamiltonian
  - Since the strong interactioins respect conservation of strangeness these are **stationary states**!

$$|K^0\rangle = \binom{1}{0} \qquad |\bar{K}^0\rangle = \binom{0}{1}$$

Base vectors in 2-dim Hilbert space

$$|\psi\rangle = \frac{1}{\sqrt{(a^2+b^2)}}(a|K^0\rangle + b|\bar{K}^0\rangle) = \frac{1}{\sqrt{(a^2+b^2)}}\binom{a}{b}$$

Time Evolution of the Kaon System

- Oh, well, unfortunately weak interaction cannot be switched off and the kaons **do decay**
- Theory offers two approaches to attack this problem
  - We could expand the 2-dim Hilbert space and take into account all the possible final states
  - or…, we could stay in the 2-dim space and introduce **effective Hamiltonian** that is responsible for the kaons disintegration
  - Usually the later option is picked up!

- Now, the leap to the Schrodinger equation describing **two state** system with the effective Hamiltonian is done by noticing that we no longer deal with **stationary** states – they can **decay**
- The consequence is that the Hamiltonian is no longer a **Hermitian operator** – the probability is no longer conserved for decaying states!

Effective Hamiltonian

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = \mathcal{H}_{eff} |\psi(t)\rangle$$

Complex 2×2 matrix

$$\mathcal{H}_{eff} = \mathcal{M} - \frac{i}{2}\Gamma$$

$$\mathcal{M} = \frac{1}{2}\left(\mathcal{H}_{eff} + \mathcal{H}_{eff}^{\dagger}\right), \Gamma = i\left(\mathcal{H}_{eff} - \mathcal{H}_{eff}^{\dagger}\right)$$

$$\mathcal{M} = \mathcal{M}^{\dagger} \rightarrow \mathcal{M}_{ij} = \mathcal{M}_{ji}^{*}$$

Both Hermitian

$$\Gamma = \Gamma^{\dagger} \rightarrow \Gamma_{ij} = \Gamma_{ji}^{*}$$

- **Mass matrix** – its e-values represents masses of the states in their CM frame (real parts of the energy levels)
- **Decay matrix** – introduced to describe decay characteristics of the system

Effective Hamiltonian

- The main purpose here is to provide explicit form of the $\mathcal{H}_{eff}$, and one can start from writing down the $\mathcal{H}_{eff}$ matrix in the most generic form

$$\mathcal{H}_{eff} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\langle K^0 | \mathcal{H}_{eff} | K^0 \rangle = A \qquad \langle \bar{K}^0 | \mathcal{H}_{eff} | \bar{K}^0 \rangle = D = A$$

$\mathcal{CPT}$ theorem states that the masses of K^0 and $\bar{K}^0$ must be the same

$$\mathcal{H}_{eff} = \begin{pmatrix} A & B \\ C & A \end{pmatrix}$$

The most generic form of the $\mathcal{H}_{eff}$ consistent with $\mathcal{CPT}$ theorem

- Next, let's express the e-states of the **effective** Hamiltonian in terms of our base states of the strong interactions

Effective Hamiltonian and its e-states

$$|K_S^0\rangle = \frac{1}{\sqrt{(|p|^2+|q|^2)}}(p|K^0\rangle + q|\overline{K}^0\rangle) = \frac{1}{\sqrt{(|p|^2+|q|^2)}}\binom{p}{q}$$

$$|K_L^0\rangle = \frac{1}{\sqrt{(|r|^2+|s|^2)}}(r|K^0\rangle + s|\overline{K}^0\rangle) = \frac{1}{\sqrt{(|r|^2+|s|^2)}}\binom{r}{s}$$

p, q, r, s are complex numbers defining the decomposition of K_S^0 and K_L^0

- e-states of the effective Hamiltonian, K_S^0 and K_L^0, have e-values in their CM frame as follow:

masses of the e-states

$$m_S - \frac{i}{2}\gamma_S,\ m_L - \frac{i}{2}\gamma_L$$

widths of the e-states

$$\mathcal{H}_{eff}\left|K_S^0\right\rangle = \left(m_S - \frac{i}{2}\gamma_S\right)\left|K_S^0\right\rangle \quad (*)$$

$$\mathcal{H}_{eff}|K_L^0\rangle = \left(m_L - \frac{i}{2}\gamma_L\right)|K_L^0\rangle$$

Effective Hamiltonian and its e-states

- Now, in the basis of K_S^0 and K_L^0 e-states, the diagonal elements of the effective Hamiltonian are as above
- We can relate them to the diagonal elements of the same operator expressed in the K^0 and $\overline{K}^0$ basis using the **trace of matrix** (trace is invariant w.r.t. base transformations)

$$Tr(\mathcal{H}_{eff}) = 2A = \left(m_S - \frac{i}{2}\gamma_S\right) + \left(m_L - \frac{i}{2}\gamma_L\right)$$

$$A = \frac{1}{2}(m_S + m_L) - \frac{i}{4}(\gamma_S + \gamma_L)$$

- Now, rewrite the equation (*)

$$\begin{pmatrix} A & B \\ C & A \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \left(m_S - \frac{i}{2}\gamma_S\right)\begin{pmatrix} p \\ q \end{pmatrix}$$

$$\begin{pmatrix} A - m_S + \frac{i}{2}\gamma_S & B \\ C & A - m_S + \frac{i}{2}\gamma_S \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = 0$$

A system of coupled linear homogenous equations!

Effective Hamiltonian and its e-states

- Non trivial solution exists only if:

$$det\begin{pmatrix} A - m_S + \frac{i}{2}\gamma_S & B \\ C & A - m_S + \frac{i}{2}\gamma_S \end{pmatrix} = 0$$

$$BC = \left(A - m_S + \frac{i}{2}\gamma_S\right)^2 = \left[\frac{1}{2}(m_S + m_L) - \frac{i}{4}(\gamma_S + \gamma_L)\right]^2$$

$$\pm\sqrt{BC} = \frac{1}{2}(m_S + m_L) - \frac{i}{4}(\gamma_S + \gamma_L)$$

- Substituting to equations describing short and long states respectively one can get:

$$\boxed{\frac{p}{q} = \pm\sqrt{\frac{B}{C}}, \frac{r}{s} = \mp\sqrt{\frac{B}{C}} = -\frac{p}{q}}$$

$$\boldsymbol{r = p, s = -q}$$

Effective Hamiltonian and its e-states

- So, the e-states of the effective Hamiltonian are:

$$|K_S^0\rangle = \frac{1}{\sqrt{(|p|^2 + |q|^2)}}(p|K^0\rangle + q|\bar{K}^0\rangle)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{(|p|^2 + |q|^2)}}(p|K^0\rangle - q|\bar{K}^0\rangle)$$

- We can now express the parameters p, q in terms of ϵ

$$\mathrm{p} = 1 + \epsilon, q = -(1 - \epsilon)$$

- And the strong e-states can be written as:

$$|K^0\rangle = \frac{\sqrt{(|p|^2 + |q|^2)}}{2p}\left(|K_S^0\rangle + |K_L^0\rangle\right)$$

$$|\bar{K}^0\rangle = \frac{\sqrt{(|p|^2 + |q|^2)}}{2q}\left(|K_S^0\rangle - |K_L^0\rangle\right)$$

Effective Hamiltonian and its e-states

- K_S^0 and K_L^0 are the e-states of the $\mathcal{H}_{eff}$, thus, the solutions of our Schrodinger equation are

$$\boxed{|K_S^0(t)\rangle = e^{-\frac{i}{\hbar}\left(m_S - \frac{i}{2}\gamma_S\right)t}|K_S^0\rangle}$$

$$\boxed{|K_L^0(t)\rangle = e^{-\frac{i}{\hbar}\left(m_L - \frac{i}{2}\gamma_L\right)t}|K_L^0\rangle}$$

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = \mathcal{H}_{eff}|\psi(t)\rangle$$

- These states decay with the lifetimes

$$\tau_S = \frac{\hbar}{\gamma_S} = 0.9 \times 10^{-10}\ s \qquad \tau_L = \frac{\hbar}{\gamma_L} = 5.0 \times 10^{-8}\ s$$

- Note! Unlike K^0 and $\overline{K}^0$ the e-states of $\mathcal{H}_{eff}$ - K_S^0 and K_L^0 are not each other's antiparticle! Thus, $m_S \neq m_L$ and $\tau_S \neq \tau_L$
- **Awesome**!

Time Evolution Final!

- Finally we are able to write down equations that **govern the time** evolution of kaons, let's assume that we start with a pure beam of K^0

$$|K^0(t)\rangle = \frac{\sqrt{(|p|^2+|q|^2)}}{2p}\left(|K_S^0(t)\rangle + |K_L^0(t)\rangle\right) =$$

$$\frac{\sqrt{(|p|^2+|q|^2)}}{2p}\left(e^{-\frac{i}{\hbar}\left(m_S-\frac{i}{2}\gamma_S\right)t}|K_S^0\rangle + e^{-\frac{i}{\hbar}\left(m_L-\frac{i}{2}\gamma_L\right)t}|K_L^0\rangle\right) =$$

$$\frac{\sqrt{(|p|^2+|q|^2)}}{2p}\left[e^{-\frac{i}{\hbar}\left(m_S-\frac{i}{2}\gamma_S\right)t}\frac{1}{\sqrt{(|p|^2+|q|^2)}}(p|K^0\rangle + q|\bar{K}^0\rangle) + e^{-\frac{i}{\hbar}\left(m_L-\frac{i}{2}\gamma_L\right)t}\frac{1}{\sqrt{(|p|^2+|q|^2)}}(p|K^0\rangle - q|\bar{K}^0\rangle)\right] =$$

$$\frac{1}{2p}\left[p\left(e^{-\frac{i}{\hbar}\left(m_S-\frac{i}{2}\gamma_S\right)t} + e^{-\frac{i}{\hbar}\left(m_L-\frac{i}{2}\gamma_L\right)t}\right)|K^0\rangle\right] +$$

$$+\frac{1}{2p}\left[p\left(e^{-\frac{i}{\hbar}\left(m_S-\frac{i}{2}\gamma_S\right)t} - e^{-\frac{i}{\hbar}\left(m_L-\frac{i}{2}\gamma_L\right)t}\right)|\bar{K}^0\rangle\right]$$

Time Evolution Final!

- So, the probability of finding K^0 in the beam at some time t is:

$$P(K^0,t) = |\langle K^0|K^0(t)\rangle|^2 = \frac{1}{4}\left|e^{-\frac{i}{\hbar}\left(m_S-\frac{i}{2}\gamma_S\right)t} + e^{-\frac{i}{\hbar}\left(m_L-\frac{i}{2}\gamma_L\right)t}\right|^2$$
$$= \frac{1}{4}\left(e^{-\frac{\gamma_S t}{\hbar}} + e^{-\frac{\gamma_L t}{\hbar}} + e^{-\frac{1}{2\hbar}(\gamma_S+\gamma_L)t} \times 2cos(m_L - m_S)\frac{t}{\hbar}\right)$$
$$= \frac{1}{4}e^{-\frac{t}{\tau_S}} + \frac{1}{4}e^{-\frac{t}{\tau_L}} + \frac{1}{2}e^{-\left(\frac{1}{\tau_S}+\frac{1}{\tau_S}\right)t} cos\frac{\Delta m t}{\hbar}$$

- And by analogy one can calculate the same for $\overline{K}^0$

$$P(\overline{K}^0,t) = |\langle \overline{K}^0|K^0(t)\rangle|^2$$
$$= \left|\frac{q}{p}\right|^2\left[\frac{1}{4}e^{-\frac{t}{\tau_S}} + \frac{1}{4}e^{-\frac{t}{\tau_L}} - \frac{1}{2}e^{-\left(\frac{1}{\tau_S}+\frac{1}{\tau_S}\right)t} cos\frac{\Delta m t}{\hbar}\right]$$

$$\Delta m = m_L - m_S \approx 3.5 \times 10^{-12}\ MeV$$

Mass difference is not zero!!

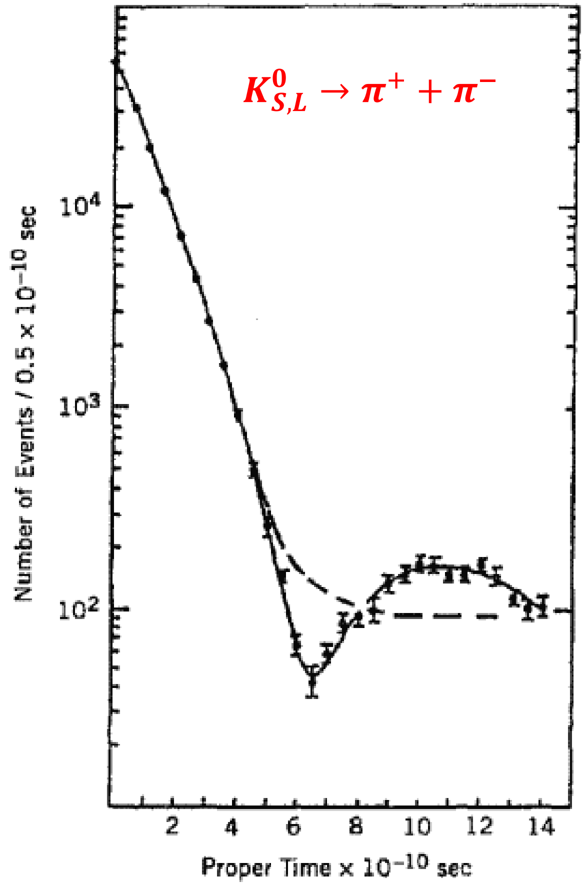
Time Evolution Final!

- The mass splitting of the weak Hamiltonian e-states can be translated into mass splitting of the strong interactions

$$m_{K^0} - m_{\overline{K}^0} < 10^{-18}\, m_{K^0}$$

- Very precise test of $\mathcal{CPT}$ symmetry

- Using our theoretical framework we could also estimate the prob. of observing weak e-states in the beam as a function of time
- By studying the number of decays as a function of the proper time one can observe QM interference in the 2π decay modes of the K_S^0 and K_L^0



Next time…

- Analogical calculations can be done for beauty mesons
- We are going to derive selected results presented today during our tutorial sessions (2 or 3 weeks time)