

# CP Violation In Heavy Flavour Physics

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# The CKM Matrix

1. In two generation system (1964) – one angle – no CP violation
2. Third generation proposed by Kobayashi & Maskawa (1973) opened the Pandora's box of new ideas how to measure CPV.
3. Many possible parametrizations:

a) original K-M:  $s_i = \sin \vartheta_i$ ;  $c_i = \cos \vartheta_i$

b) Standard representation (PDG proposal):

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{23} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \end{pmatrix}$$

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \underbrace{\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

$$\begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{bmatrix}$$

The CKM matrix is described by three rotation angles and a complex phase.

# CKM matrix parametrization

## 4. Wolfenstein parametrization (1983):

- matrix elements are expanded in terms of  $\sin \vartheta_i \equiv \lambda$ ;
- from kaons sector we have  $V_{us} = \lambda = 0.22$ , and  $V_{ud} = (1 - \lambda^2/2)$
- from B-lifetime:  $V_{cb} = 0.04 - 0.006 = A\lambda^2$
- let's keep  $V_{ud}, V_{us}, V_{tb}$  real – expressed in term of four real parameters :  $\lambda, A, \rho, \eta$
- the only complex components are in  $V_{ub}$  and  $V_{td}$ , third row/column are of order smaller (CPV effects)  $A\lambda^3(\rho - i\eta)$
- For the CP violation – one phase should be measurable, so  $\eta$  cannot be zero

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

$$\begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta + i\eta\frac{\lambda^2}{2}) \\ -\lambda & 1 - \frac{\lambda^2}{2} - i\eta A\lambda^4 & A\lambda^2(1 + i\eta\lambda^2) \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$|V_{td}|e^{-i\beta}$$

$$|V_{ub}|e^{-i\gamma}$$

# Higher order in CKM

5. Higher order in Wolfenstein parametrization:

$$\begin{pmatrix} 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda - A^2\lambda^5(\rho + i\eta - \frac{1}{2}) & 1 - \frac{\lambda^2}{2} - (\frac{1}{8} + \frac{A}{2})\lambda^4 & A\lambda^2 \\ A\lambda^3[1 - (\rho + i\eta)(1 - \frac{\lambda^2}{2})] & -A\lambda^2 - A\lambda^4(\rho + i\eta - \frac{1}{2}) & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

$$|V_{ts}|e^{-i\beta_s}$$

$\beta$  and  $\beta_s$  are weak phases in  $B^0$  and  $B_s^0$  mixing,  
 $\beta$  and  $\gamma$  are the CKM angles (see next slides).  
 They are ones of the most important observables in experimental heavy flavour physics

# Unitarity of CKM matrix

The CKM matrix is unitary  $V_{CKM}^{-1} = V_{CKM}^\dagger$  – this give us 12 orthogonality conditions:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$$

$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0$$

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

$$V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} = 0$$

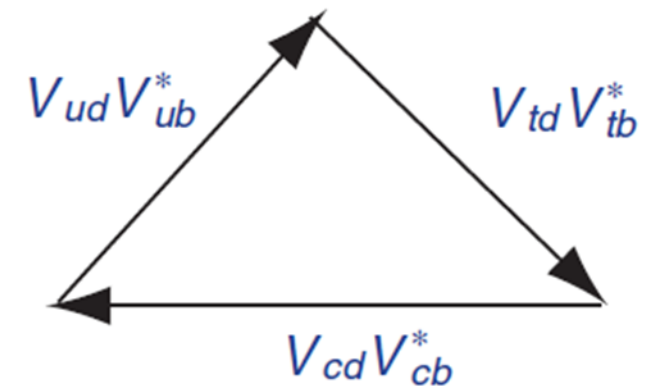
$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0$$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

The orthogonality conditions can be regarded as a triangle condition – CKM matrix elements are complex numbers, so their sum is simply a sum of three vectors:



# Unitarity of CKM matrix

But most of them have magnitudes of very different size and are currently useless from experimental point of view :

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0 \quad \lambda, \lambda, \lambda^5$$

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0 \quad \lambda^3, \lambda^3, \lambda^3$$

$$V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} = 0 \quad \lambda^4, \lambda^2, \lambda^2$$

$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0 \quad \lambda, \lambda, \lambda^5$$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \quad \lambda^3, \lambda^3, \lambda^3$$

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0 \quad \lambda^4, \lambda^2, \lambda^2$$

The most attractive are two triangles:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

# Unitarity of CKM matrix

But still two promising left:

$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0$	$\lambda, \lambda, \lambda^5$
$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$	$\lambda^3, \lambda^3, \lambda^3$
$V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} = 0$	$\lambda^4, \lambda^2, \lambda^2$
$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$	$\lambda, \lambda, \lambda^5$
$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$	$\lambda^3, \lambda^3, \lambda^3$
$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0$	$\lambda^4, \lambda^2, \lambda^2$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

„The” unitary triangle!

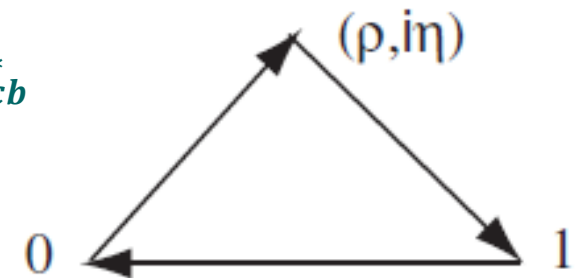
Using Wolfenstein parametrization, we can draw them on complex plane :

$$V_{ud} V_{ub}^* = A\lambda^3(1 - \lambda^2/2)(\rho + i\eta)$$

$$V_{cd} V_{cb}^* = -A\lambda^3$$

$$V_{td} V_{tb}^* = A\lambda^3(1 - \rho - i\eta)$$

if sides are divided by  $V_{cd} V_{cb}^*$   
the UT looks like that:



# The unitary triangle

Try your vector algebra...

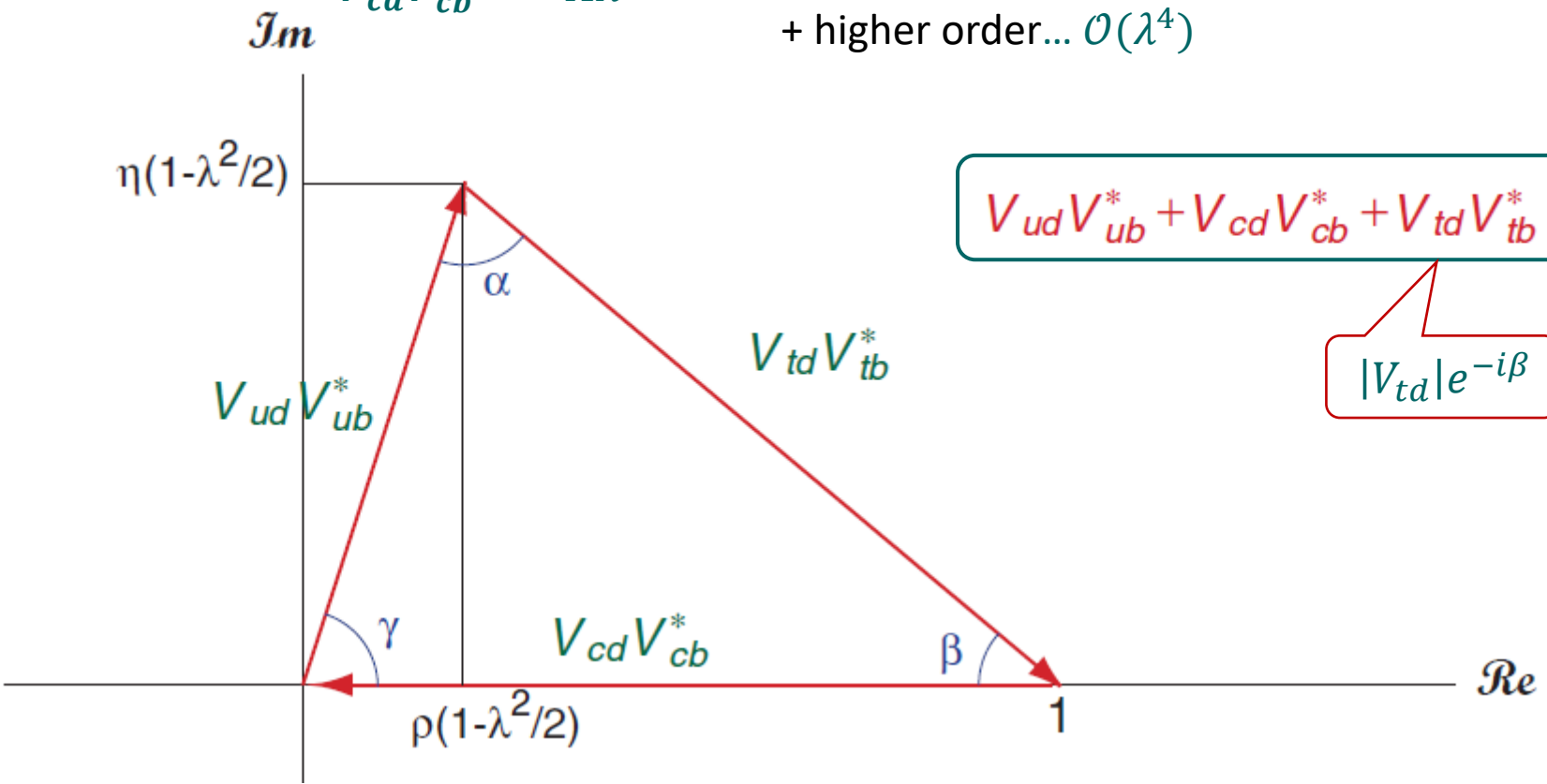
$$V_{ud}V_{ub}^* = A\lambda^3(1 - \lambda^2/2)(\rho + i\eta)$$

$$V_{td}V_{tb}^* = A\lambda^3(1 - \rho - i\eta)$$

$$V_{cd}V_{cb}^* = -A\lambda^3$$

+ higher order...  $\mathcal{O}(\lambda^4)$

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$|V_{td}| e^{-i\beta}$$

$$\alpha = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\beta = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$



# And another unitarity triangle

and more complex example...

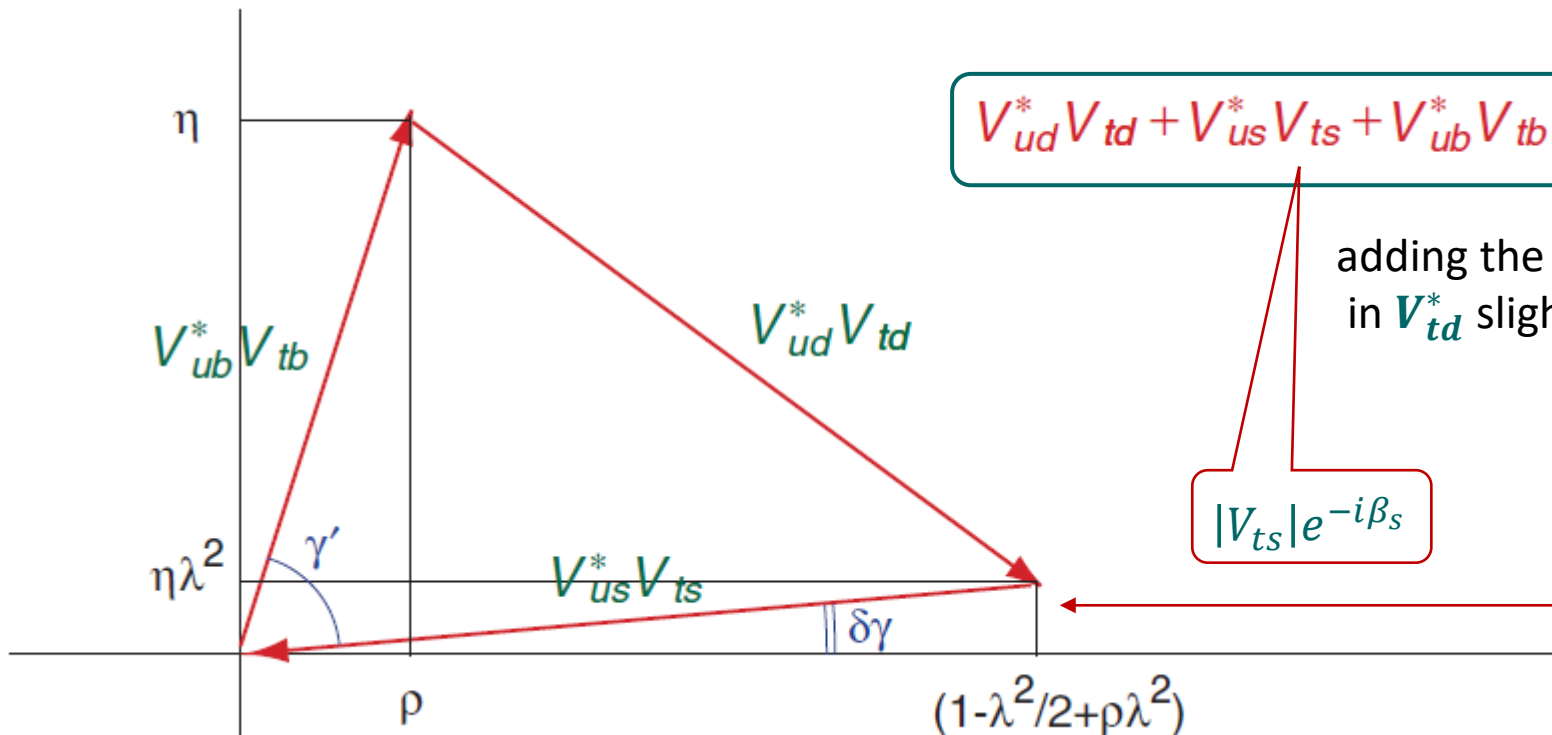
$$V_{ub}V_{tb}^* = A\lambda^3(\rho + i\eta)$$

$$V_{ud}V_{td}^* = A\lambda^3(1 - \lambda^2/2)(1 - \rho - i\eta)$$

$$V_{us}V_{ts}^* = -A\lambda^3$$

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

+ higher order...  $\mathcal{O}(\lambda^4)$



$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

adding the term  $\mathcal{O}(\lambda^4)$  in  $V_{td}^*$  slightly tilts the  $\Delta$

$$|V_{ts}|e^{-i\beta_s}$$

$$-iA\lambda^4\eta$$

precise measurements can prove this!

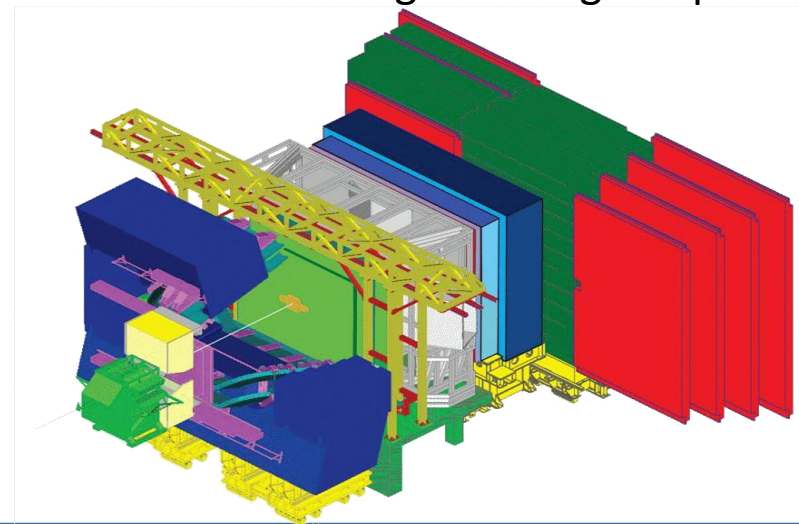
1. HEP has always two main aims:
  - to confirm the SM
  - or**
  - to find evidences for Physics Beyond the Standard Model.
2. Precise measurement of the UTs are able to fulfill both....
  - a) If the triangle remains triangular – we have three generation of quarks with small CP violation effects.
  - b) If one angle is „open”- fourth generation?
  - c) If an angle is greater then predictions – new particles were exchanged?
3. So the main purpose in WI is now to over constrain the UTs – measure all sides and angles with great precision and to compare them with SM predictions.

## HOW?

With charm and beauty mesons decays.

## WHERE?

At the LHCb spectrometer.



# Sides of the Unitary Triangles

Sides of the UT can be measured with:

$V_{ud}$	$\beta$ -decay	Nuclear physics	$\cos \vartheta_i$
$V_{us}$	K decay	$K^{+0} \rightarrow \pi^{0+} l^+ \nu_l$	$\sin \vartheta_i$
$V_{cd}$	Neutrino scattering	$\nu_\mu d \rightarrow \mu^+ c$	$\cos \vartheta_i$
$V_{cs}$	Charm decay	$D_S^+ \rightarrow \mu^+ \nu_\mu$	$BR$
$V_{ub}$	B decay	$B^0 \rightarrow \pi^- e^+ \nu_e$	$BR$
$V_{cb}$	B decay to charm		
$V_{td}$	B mixing		

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$b \rightarrow u$   
transitions

$b \rightarrow c$   
transitions

$B^0$  mixing

# Angles of the Unitary Triangles

Angles of the UT can be measured with:

$$B^0 \rightarrow J/\psi K_S \quad \sin 2\beta$$

$$B^0 \rightarrow \pi^+ \pi^- \quad \sin 2\alpha$$

$$B_S^0 \rightarrow D_S^+ K^- \quad \sin 2\gamma$$

$$\text{Weak phase} \quad \beta_S$$

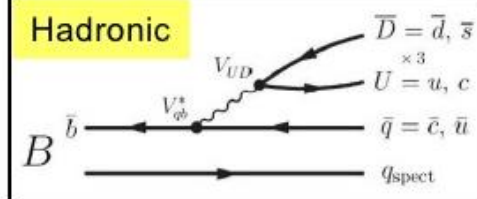
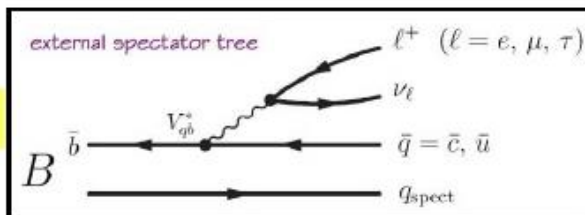
Short history of flavour physics:

1. First B physics experiments were build on symmetric electron-positron collider:
  - Petra (DESY) in 80'ties
  - **LEP at CERN in 1994-2000**
2. Then two asymmetric B-factories (currently not taking data):
  - Belle (Japan)
  - BaBar (SLAC,USA)
3. LHC
  - **LHCb – dedicated B physics experiment**
  - CMS, ATLAS also interested in heavy flavours

# Market with diagrams

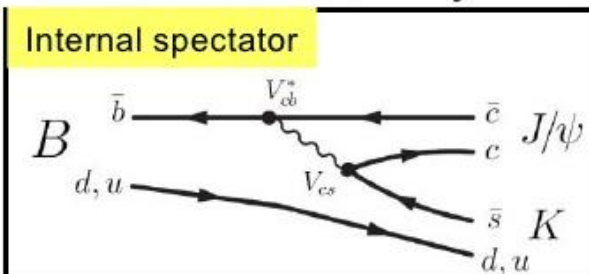
## Dominant decays

### Semi-leptonic

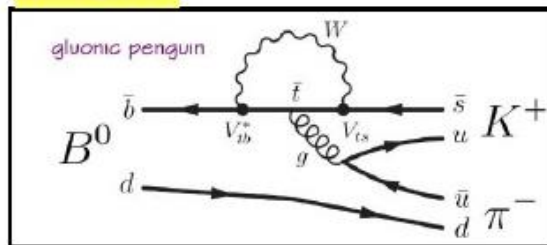


## Rare hadronic decays

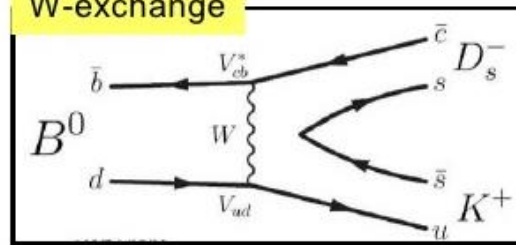
### Internal spectator



### Gluonic penguin

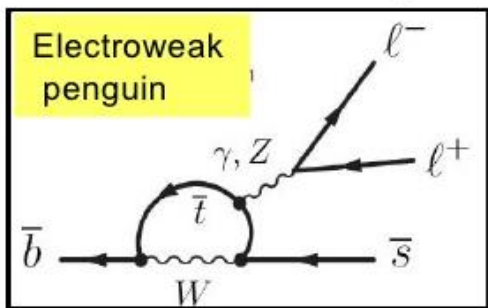


### W-exchange

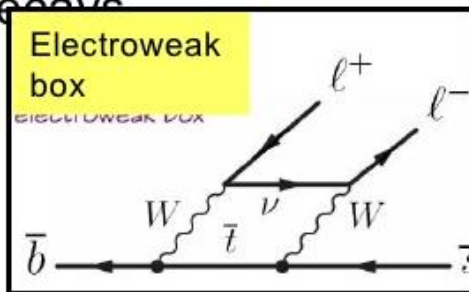


## Radiative and leptonic decays

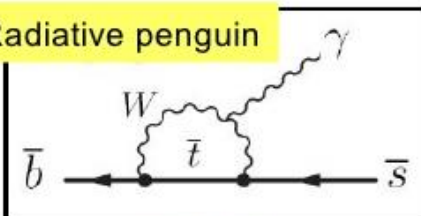
### Electroweak penguin



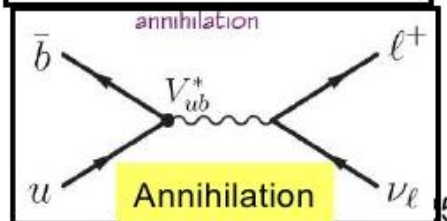
### Electroweak box



### Radiative penguin



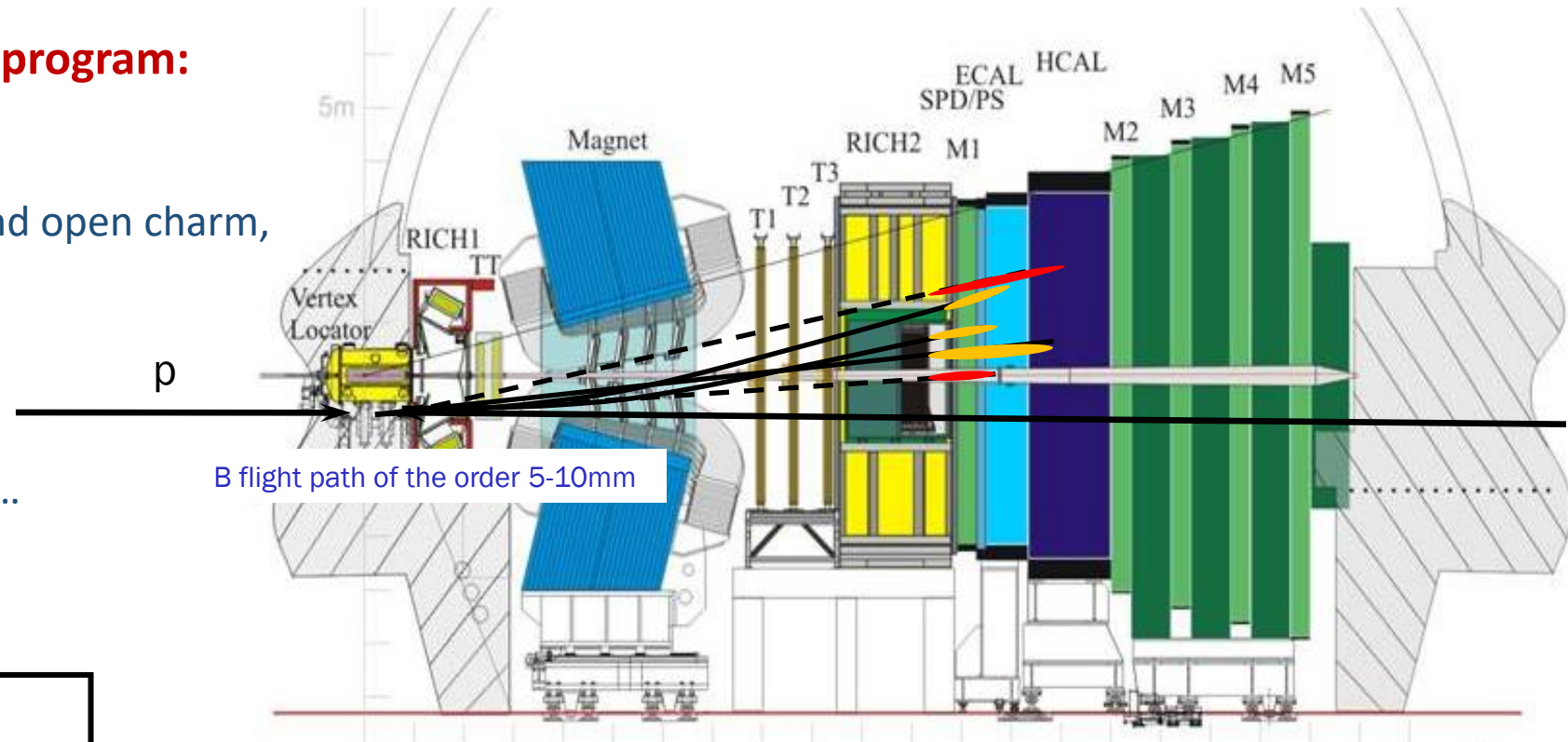
### Annihilation



Summer School KPI 15 August 2009

### Physics program:

- CP Violation ,
- Rare B decays,
- B decays to charmonium and open charm,
- Charmless B decays,
- Semileptonic B decays,
- Charm physics
- B hadron and quarkonia
- QCD, electroweak, exotica ...



Tracking:  
Silicon & Straw tubes  
Magnetic field

Vertexing:  
High precision silicon  
detectors (10 $\mu$ m position  
resolution) very close to  
collision point

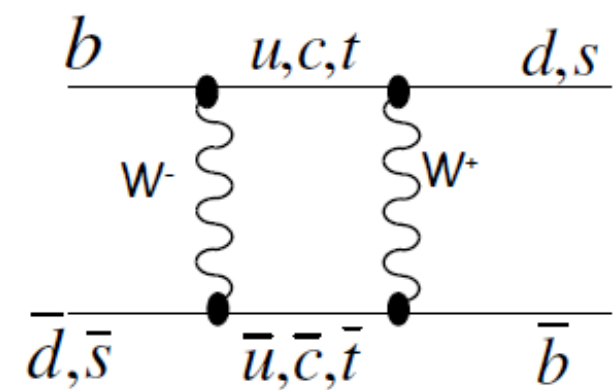
RICH performance:  
*Cherenkov radiation.*  
*Measures velocity, combine with*  
*momentum to get mass*  
*Particle identification in p range 1-100 GeV*  
 $\pi, K$  ID efficiency > 90%, misID < ~10%

Calorimeters:  
Electromagnetic &  
Hadronic calorimeters  
- Critical (with muons) for triggering

# Mixing of $B^0$ and $B_S^0$ meson

1. Like neutral kaon system, neutral B mesons may also oscillate:  $\begin{pmatrix} B^0 = d\bar{b} \\ \bar{B}^0 = \bar{d}b \end{pmatrix}$
2. The top quark transition has the dominant amplitude:  $\begin{pmatrix} B_S^0 = s\bar{b} \\ \bar{B}_S^0 = \bar{d}s \end{pmatrix}$

$$A \propto \sum \text{all pair of quarks } A_{bi} A_{jb}^*$$



	$B^0 = d\bar{b} \quad \bar{B}^0 = \bar{d}b$	$B_S^0 = s\bar{b} \quad \bar{B}_S^0 = \bar{d}s$
Oscillations parameter	$x_d = \frac{\Delta m_d}{\Gamma_d} \approx 0.72$	$x_s = \frac{\Delta m_s}{\Gamma_s} \approx 24$
Large mass difference	$\Delta m_d \approx 3.3 \cdot 10^{-13} \text{ GeV}$ $\approx 0.5 \text{ ps}^{-1}$	$\Delta m_s \approx 17.8 \text{ ps}^{-1}$
Small lifetime difference	$x_d = \frac{\Delta \Gamma_d}{\Gamma_d} \approx 5 \cdot 10^{-3}$	$x_d = \frac{\Delta \Gamma_s}{\Gamma_s} \approx 0.1$
$\frac{q}{p}$ - sensitivity to weak phase	$\frac{q}{p} = \frac{V_{td} V_{tb}^*}{V_{tb} V_{td}^*} \sim \beta$	$\frac{q}{p} = \frac{V_{ts} V_{tb}^*}{V_{tb} V_{ts}^*} \sim \beta_s$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^*}{M_{12}}}$$



# Mixing of $B^0$ and $B_S^0$ meson

1. The weak B-meson states are a combination of flavour states:

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle \quad |B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

2. In terms of the CKM elements  $q/p$  is given by:

$$\frac{q}{p} = \frac{V_{td}V_{tb}^*}{V_{tb}V_{td}^*} = e^{-i2\beta}$$

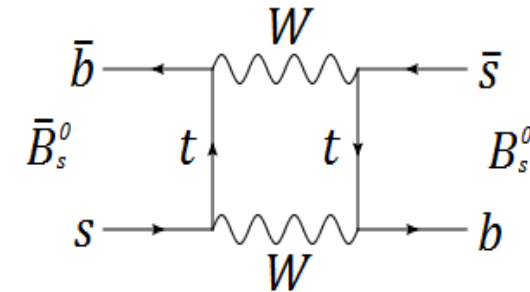
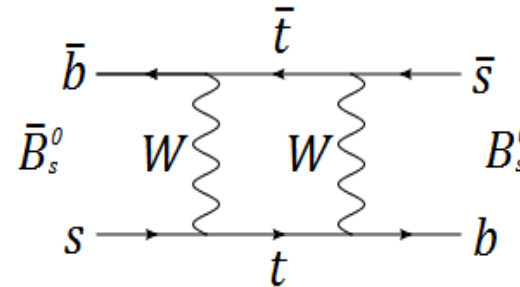
here  $d$  is replaced by  $s$  in case of  $B_S^0$

$$\frac{q}{p} = \frac{V_{ts}V_{tb}^*}{V_{tb}V_{ts}^*} = e^{-i2\beta_S}$$

so now the physical states are written as:

$$|B_L\rangle = 1/\sqrt{2} [ |B^0\rangle + e^{-i2\beta} |\bar{B}^0\rangle ]$$

$$|B_H\rangle = 1/\sqrt{2} [ |B^0\rangle - e^{-i2\beta} |\bar{B}^0\rangle ]$$



the eigenstates of the effective Hamiltonian, with definite mass and lifetime, are mixtures of the flavour eigenstates

and  $\beta$  is also called the  **$B^0$  mixing phase**

3. The states  $B_L$  and  $B_H$  are lighter and heavier state, with almost identical lifetimes:  $\Gamma_L = \Gamma_H \equiv \Gamma$

4. The mass difference  $\Delta m$  between them is greater than in kaons.



# Mixing of $B^0$ and $B_S^0$ meson

5. If we write the flavour states as a combination of weak states:

$$|B^0\rangle = 1/\sqrt{2} [|B_L\rangle + |B_H\rangle]$$

then the wavefunction evolves according to the time dependence of physical states:

$$|B(t)\rangle = 1/\sqrt{2} \{ \mathbf{a}(t) |B_L\rangle + \mathbf{b}(t) |B_H\rangle \}$$

where time dependence of coefficients is:

$$\mathbf{a}(t) = e^{-i(m_L - \frac{i}{2}\Gamma)t} \quad \mathbf{b}(t) = e^{-i(m_H - \frac{i}{2}\Gamma)t}$$

Now substitute  $\mathbf{a}(t)$  and  $\mathbf{b}(t)$  and  $|B_{L,H}\rangle$  into time-dependent wave function.

Do not forget to express mass states as a combination of flavour states....

$$|B_L\rangle = 1/\sqrt{2} [|B^0\rangle + e^{-i2\beta} |\overline{B^0}\rangle]$$

$$|B_H\rangle = 1/\sqrt{2} [|B^0\rangle - e^{-i2\beta} |\overline{B^0}\rangle]$$

# Mixing of $B^0$ and $B_S^0$ meson

6. Now substitute  $a(t)$  and  $b(t)$  and  $|B_{L,H}\rangle$  into time-dependent wave function:

$$|B(t)\rangle = 1/\sqrt{2}\{a(t)|B_L\rangle + b(t)|B_H\rangle\}$$

$$|B_L\rangle = 1/\sqrt{2} [ |B^0\rangle + e^{-i2\beta} |\overline{B^0}\rangle ]$$

$$|B_H\rangle = 1/\sqrt{2} [ |B^0\rangle - e^{-i2\beta} |\overline{B^0}\rangle ]$$

$$a(t) = e^{-i(m_L - \frac{i}{2}\Gamma)t}$$

$$b(t) = e^{-i(m_H - \frac{i}{2}\Gamma)t}$$

... and calculate the probabilities of the state to stay as a  $|B^0\rangle$

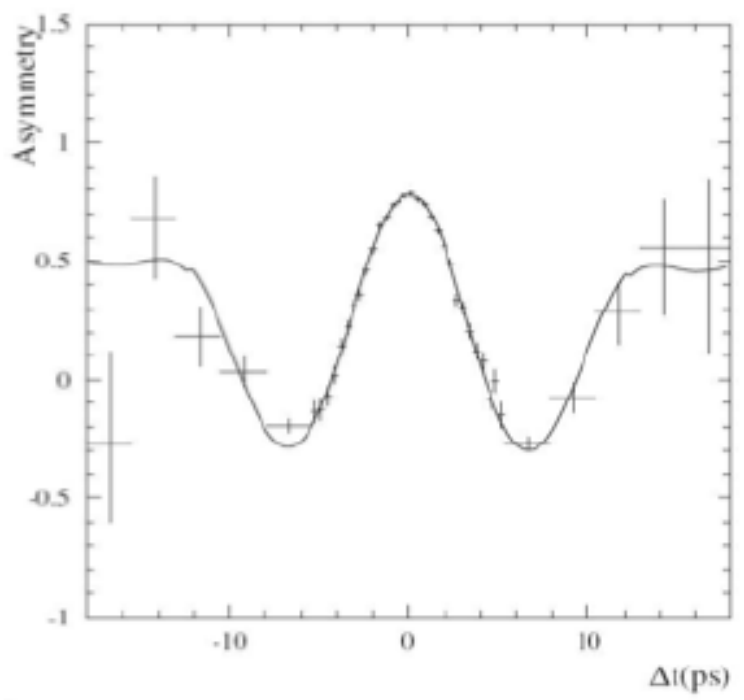
$$P(B^0(t=0) \rightarrow B^0; t) = |\langle B^0(t) | B^0 \rangle|^2 = \dots = e^{-\Gamma t} \cos^2\left(\frac{\Delta m}{2} t\right)$$

7. The same calculation can be done for  $B_S^0$

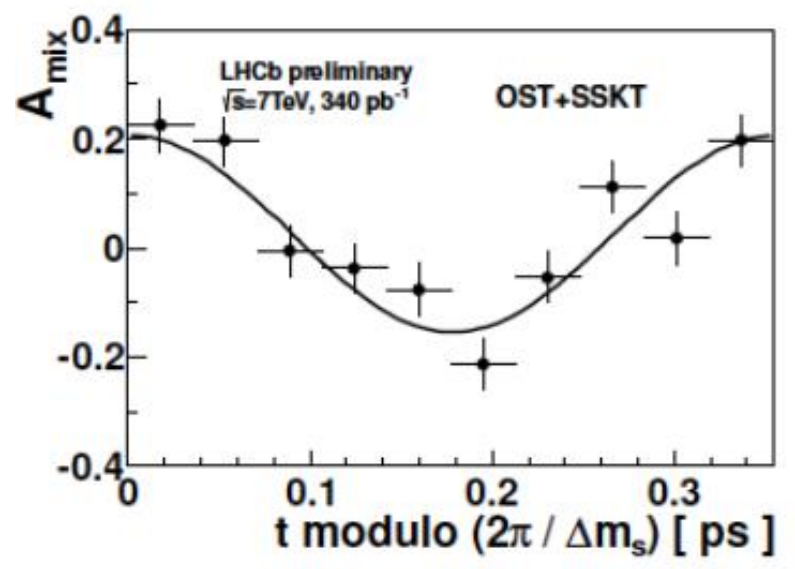
*try to do it!*

# Mixing of $B^0$ and $B_S^0$ meson

BaBar:  $\Delta m = 0.511 \pm 0.007 \text{ ps}^{-1}$



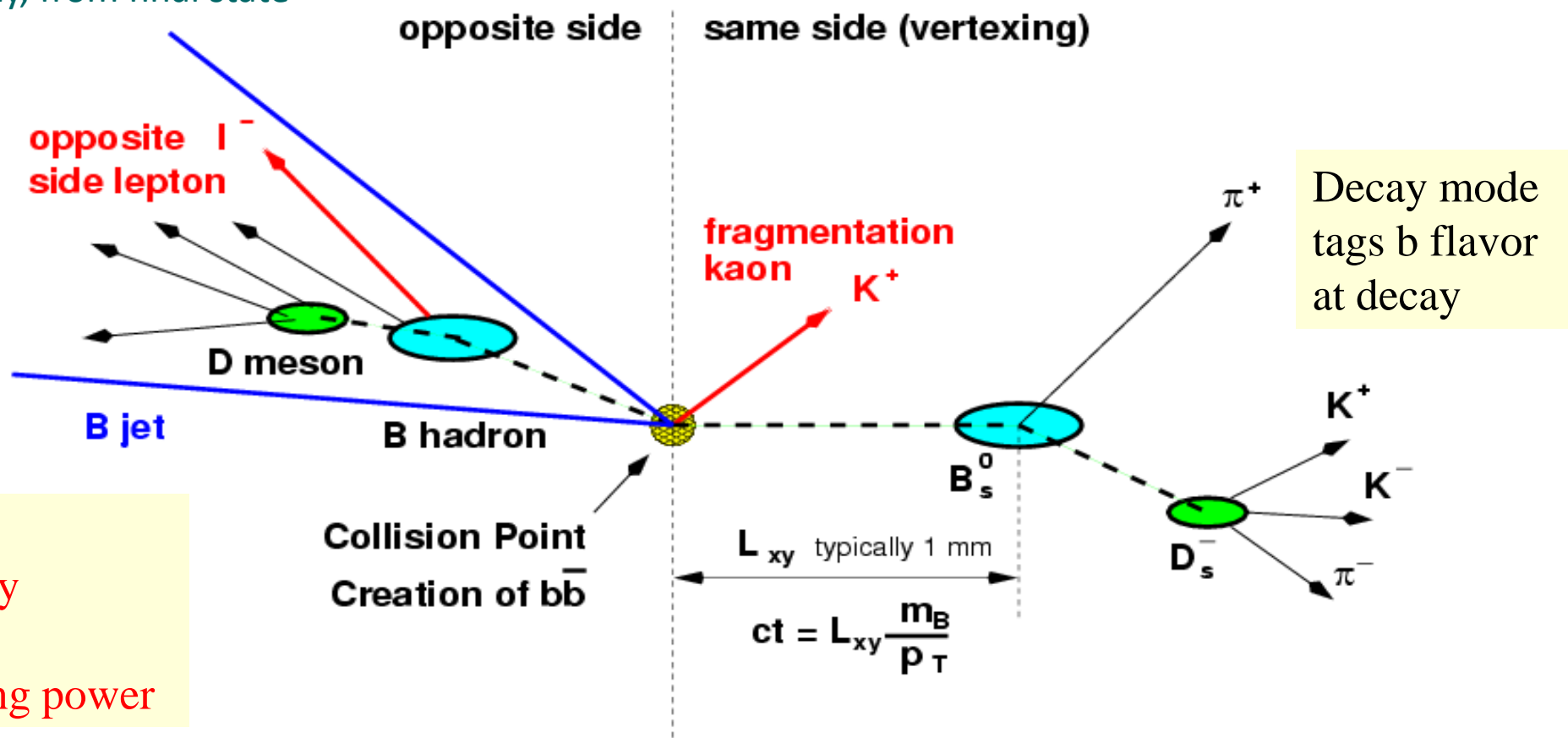
LHCb:  $\Delta m_S = 17.768 \pm 0.023 \text{ ps}^{-1}$



# Experimental challenges for mixing

1. Need to determine:

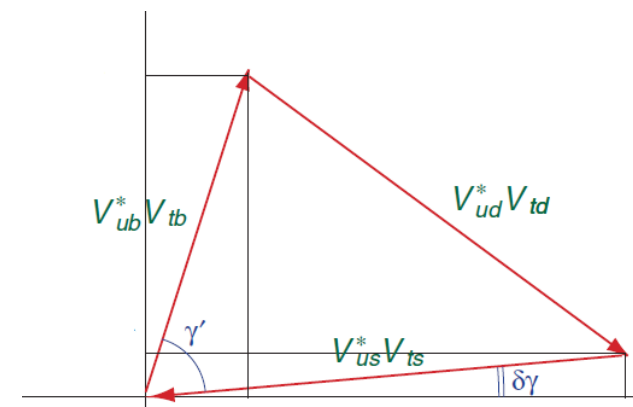
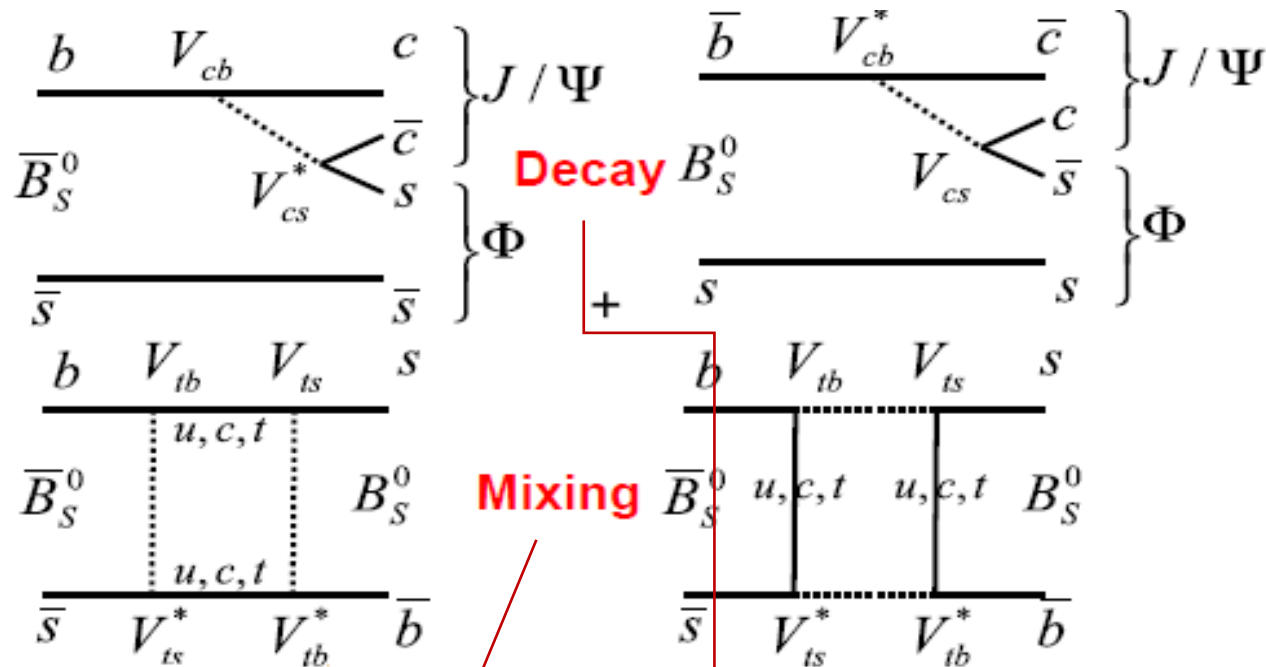
- a) Flavour at production  $\Leftrightarrow$  **tagging**
- b) Flavour at decay, from final state
- c) B decay length



Dilution  $D = 1 - 2w$   
 $w$  = mistag probability  
 $\epsilon$  = efficiency  
 $\epsilon D^2$  = effective tagging power

# Weak phase $\phi_S$

The weak phase  $\phi_S$  can be extracted from tagged Bs decays to CP eigenstates:  $B_S \rightarrow J/\psi\phi$



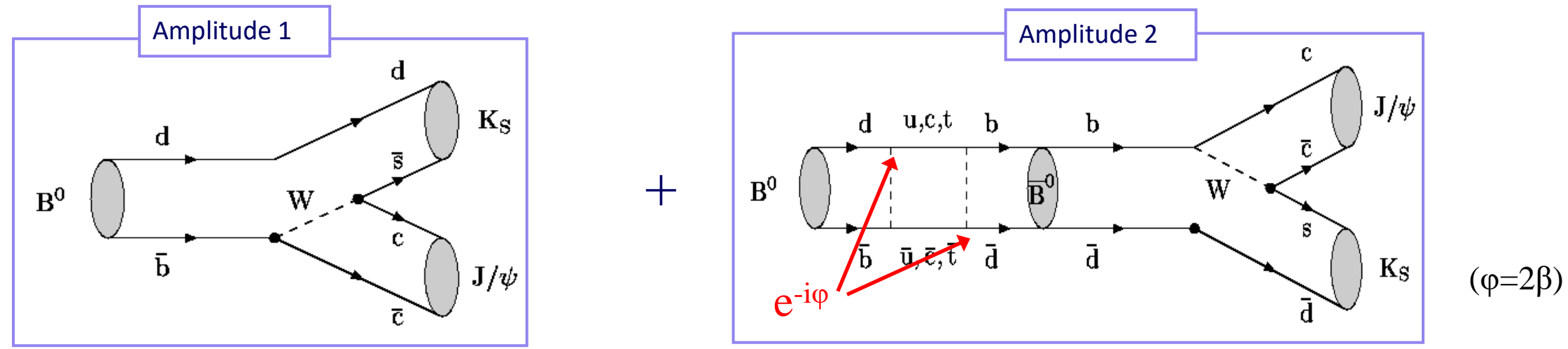
$$\lambda = \frac{q}{p} \frac{\bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}} = \left( \frac{V_{ts} V_{tb}^*}{V_{tb} V_{ts}^*} \right) \left( \frac{V_{cb} V_{cs}^*}{V_{cs} V_{cb}^*} \right) = \frac{|V_{ts}| e^{i\beta_S}}{|V_{ts}| e^{-i\beta_S}} = e^{2i\beta_S} = e^{i\phi_S}$$

$$\phi_S = -0.036 \pm 0.002$$

Very small value of  $\phi_S$  is predicted in SM. So any deviation from zero is a sign of new particle exchanged – Physics Beyond the Standard Model

# Golden channel for $\sin 2\beta$

- The process  $B^0 \rightarrow J/\psi K_S$  is called the „golden mode” for measurement of the  $\beta$  angle:
  - clean theoretical description,
  - clean experimental signature,
  - large (for a B meson) branching fraction of order  $\sim 10^{-4}$ .
- This is a process with interference of amplitudes with and without mixing:



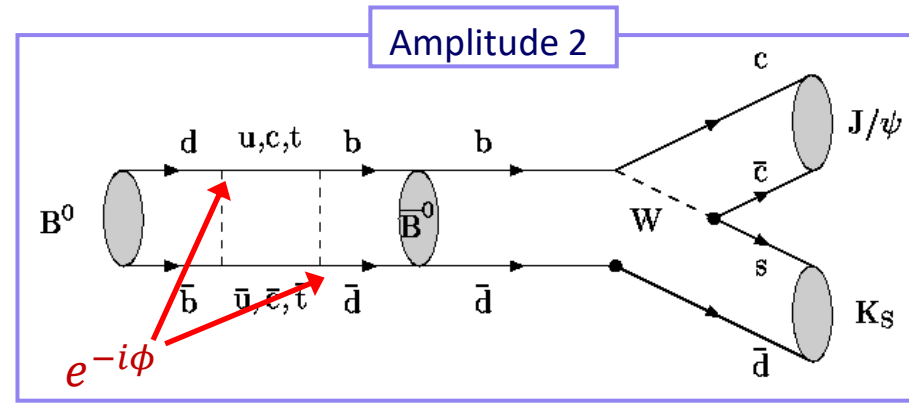
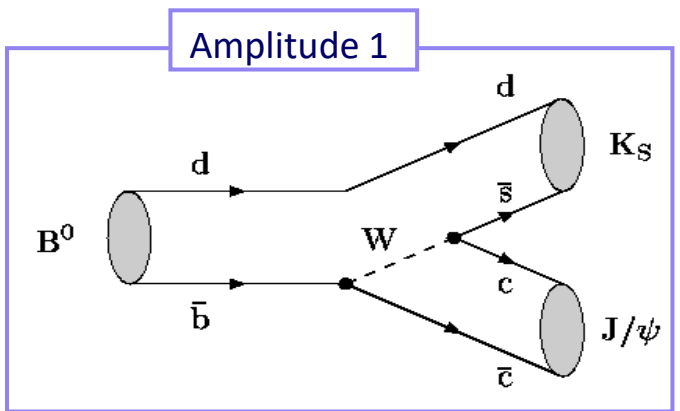
- The  $\beta$  angle sensitivity comes from the  $B^0 \leftrightarrow \bar{B}^0$  mixing due to the  $\bar{t} \rightarrow \bar{d}$  and  $t \rightarrow d$  transitions.

# Golden channel for $\sin 2\beta$

4. We need to calculate the asymmetry of the type:

$$A_{CP}(t) = \frac{\Gamma_f - \overline{\Gamma}_f}{\Gamma_f + \overline{\Gamma}_f}$$

and remember that decay rate depends on (see lect 4):  $\Gamma(B \rightarrow f) \propto |A_f|^2 = |A_1 + A_2|^2$

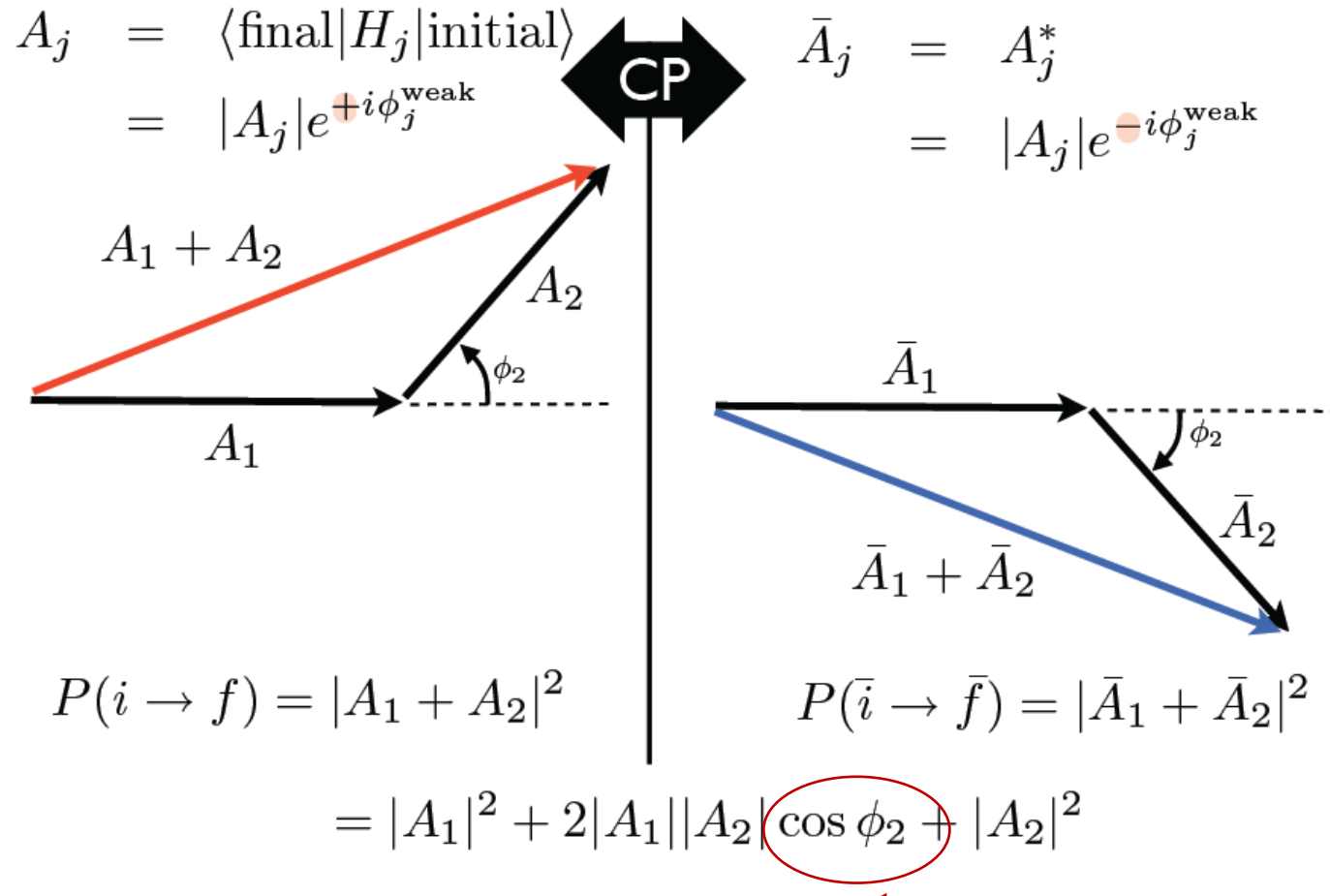


$\phi = 2\beta$

$$\Gamma(B \rightarrow J/\psi K_S) = \left| A e^{-imt - \Gamma t} \left( \cos \frac{\Delta mt}{2} + e^{-i\phi} \sin \frac{\Delta mt}{2} \right) \right|^2$$

$$A_{CP}(t) = \frac{\Gamma\{B \rightarrow J/\psi K_S\} - \Gamma\{\bar{B} \rightarrow J/\psi K_S\}}{\Gamma\{B \rightarrow J/\psi K_S\} + \Gamma\{\bar{B} \rightarrow J/\psi K_S\}} = -\sin 2\beta \sin \Delta mt$$

# Essence of amplitude interference



In case of only one decay amplitude – the decay rates are equal:

$$\Gamma(\mathbf{P} \rightarrow \mathbf{f}) = \Gamma(\bar{\mathbf{P}} \rightarrow \bar{\mathbf{f}})$$

and no CP violation occurs.

For two amplitudes the decay rates may differ and the asymmetry is sensitive to relative phase

$$A = \frac{|\bar{A}_f|^2 - |A_f|^2}{|\bar{A}_f|^2 + |A_f|^2}$$



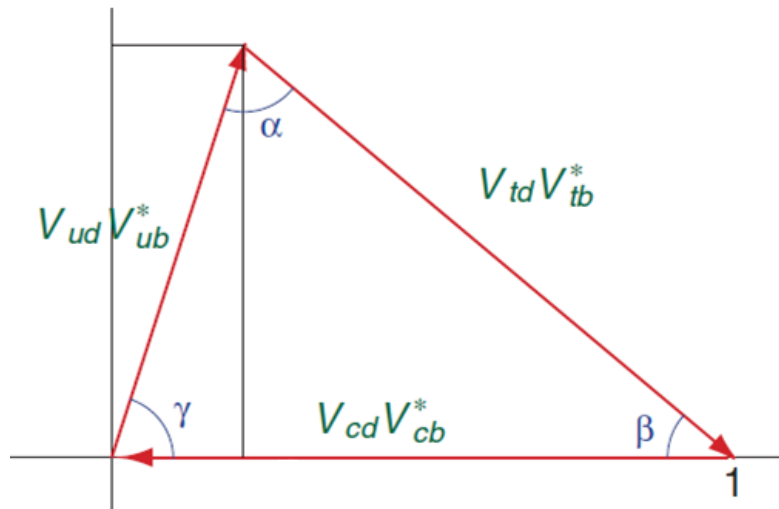
# Measurement of CKM $\gamma$ angle

1. The CKM  $\gamma$  angle can be measured through plenty of processes:

- a) time integrated decays (GLW or ADS method)
- b) time dependent CP asymmetries in transition  $b \rightarrow c\bar{u}d(s)$

2. We consider B decays of a type  $B \rightarrow DK$  with different charges and B flavours:

$$B_q \rightarrow D_q h_q$$



$$B^0 \rightarrow D^+ K^-$$

$$B^+ \rightarrow D^* K^+$$

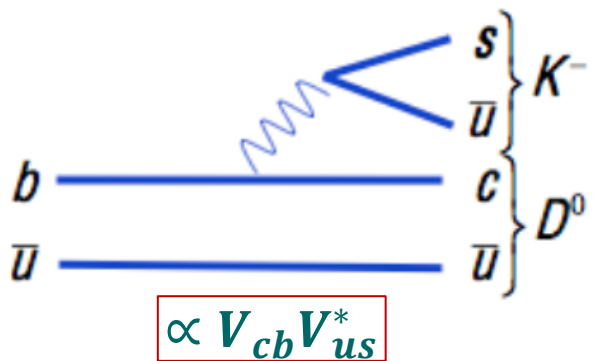
$$B_S^0 \rightarrow D_S^- K$$

$$B_S^0 \rightarrow D_S^- K^{*+}$$

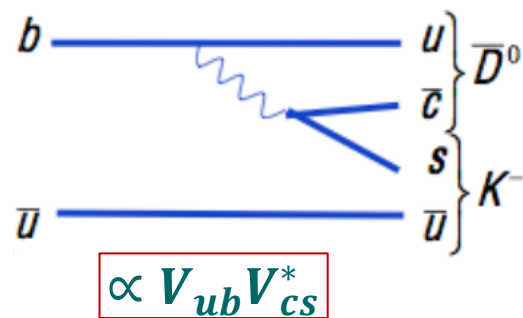
$$B_S^0 \rightarrow D_S^{*-} K^{*+}$$

# Time integrated method

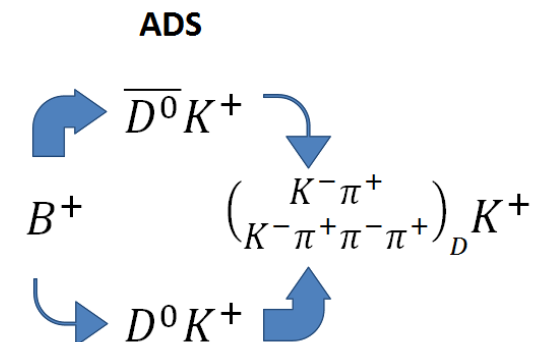
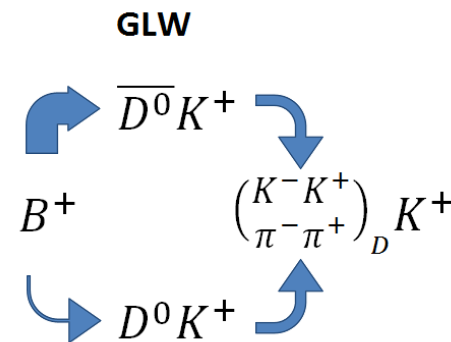
This is a measurement of angle  $\gamma$  with the processes  $B^\pm \rightarrow D^0 K^\pm$ . Plenty of methods which differ by the final states  
 Interference between two diagrams:



colour allowed



colour suppressed

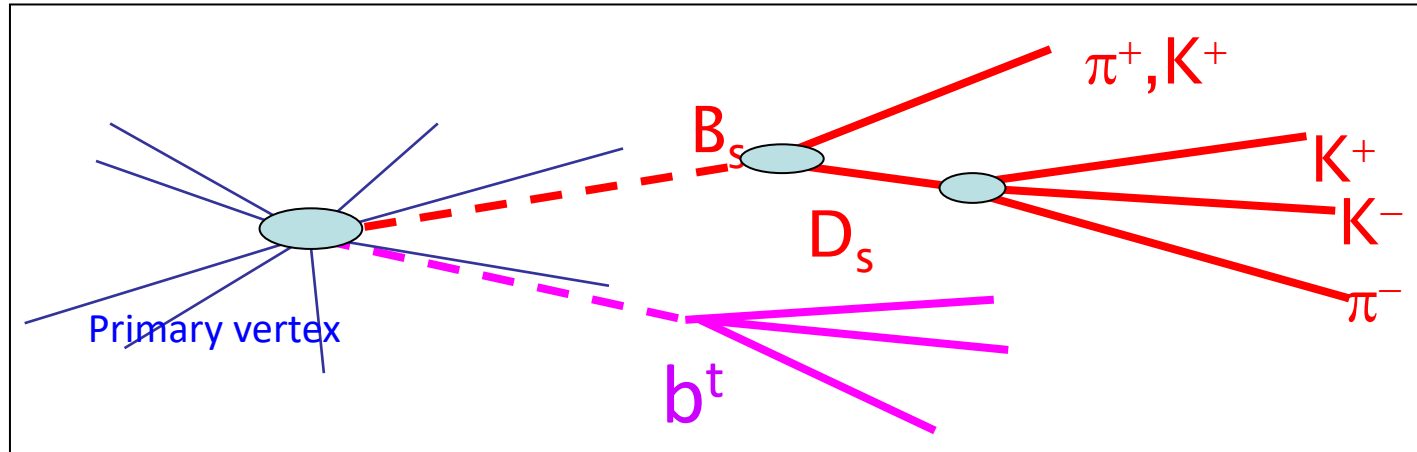


$$A_{CP} = \frac{\Gamma\{\mathbf{B}^- \rightarrow D^0 K^-\} - \Gamma\{\mathbf{B}^+ \rightarrow D^0 K^+\}}{\Gamma\{\mathbf{B}^- \rightarrow D^0 K^-\} + \Gamma\{\mathbf{B}^+ \rightarrow D^0 K^+\}} \propto \sin \gamma$$

# Time dependent $B_S^0 \rightarrow D_S^- K$

$$B_S^0 \rightarrow D_S^- K$$

This family of processes are very experimentally challenging:

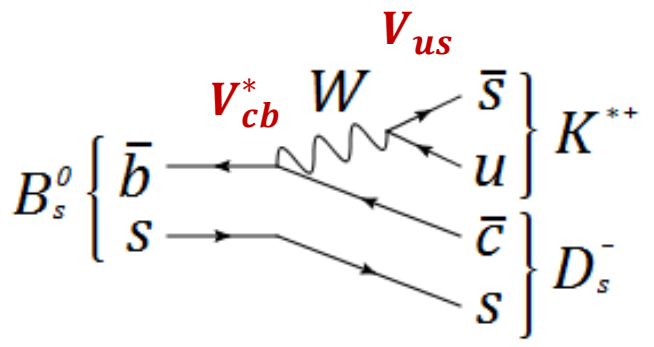
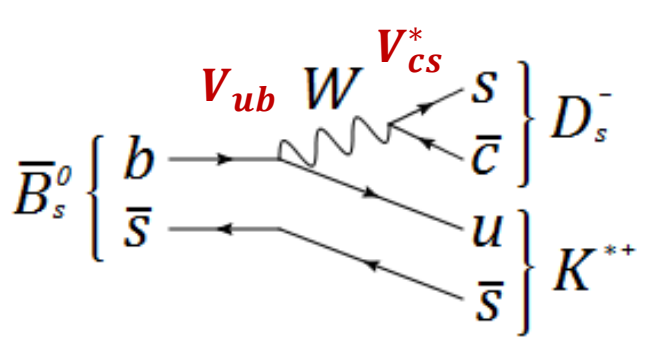


- six hadrons in the final state – very good PID and mass resolution
- high- $P_T$  tracks and displaced vertices - *efficient trigger*
- *efficient tagging and good tagging power (small mistag rate)*
- *good decay-time resolution*

# Time dependent $B_s^0 \rightarrow D_s^- K$

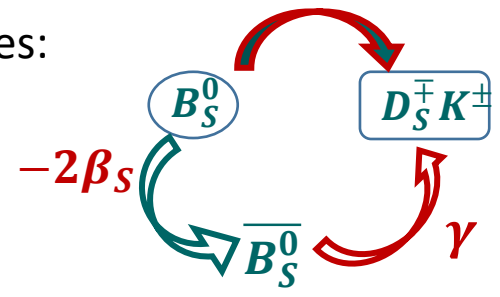
1.  $B_s^0$  and  $\overline{B}_s^0$  decay to the same final state.

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$



2.  $B_s^0$  and  $\overline{B}_s^0$  can oscillate into one another.

3. So we have interference between two processes:

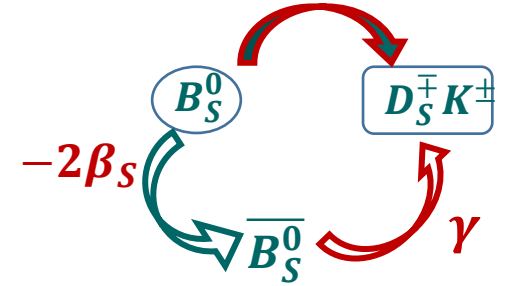


# Time dependent $B_S^0 \rightarrow D_S^- K$

We have some experience in decay rate equation...

The probability of B meson decay to final state f is given by the Fermi golden rule:

$$\Gamma_{B_S^0 \rightarrow f}(t) \sim |\langle f | T | B_S^0(t) \rangle|^2$$



and we can try to calculate it...

$$\Gamma_{B_S^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma_s t}}{2} \cdot \left( \cosh \frac{\Delta\Gamma_s t}{2} + D_f \sinh \frac{\Delta\Gamma_s t}{2} + C_f \cos \Delta m_s t - S_f \sin \Delta m_s t \right)$$

$$\Gamma_{\bar{B}_S^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma_s t}}{2} \cdot \left( \cosh \frac{\Delta\Gamma_s t}{2} + D_f \sinh \frac{\Delta\Gamma_s t}{2} - C_f \cos \Delta m_s t + S_f \sin \Delta m_s t \right)$$

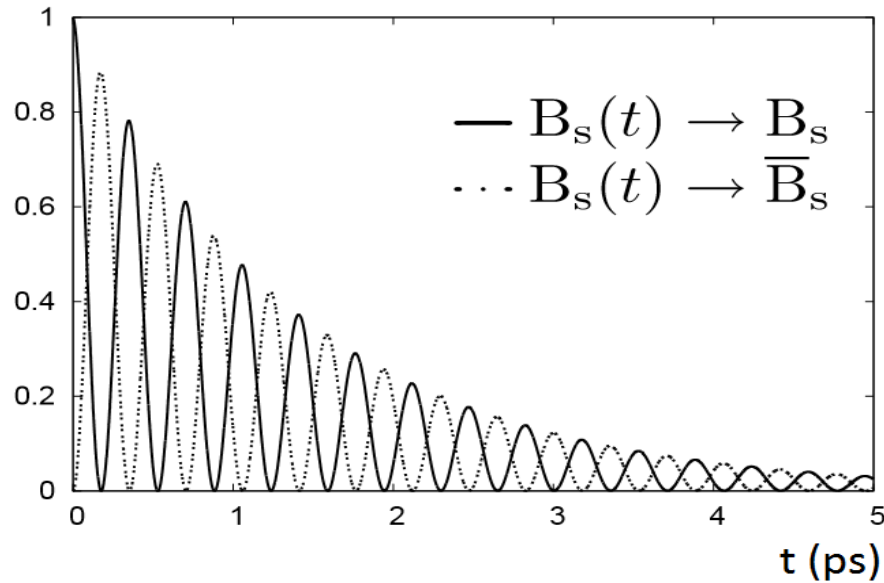
$$D_f = \frac{2\text{Re}\lambda_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\text{Im}\lambda_f}{1 + |\lambda_f|^2}$$

$$\lambda_f \equiv \frac{1}{\bar{\lambda}_f} = \frac{q \bar{A}_f}{p A_f} \quad A_f = \langle f | T | B_S^0 \rangle \quad \bar{A}_{\bar{f}} = \langle \bar{f} | T | \bar{B}_S^0 \rangle$$

good luck!

# Time dependent $B_s^0 \rightarrow D_s^- K$

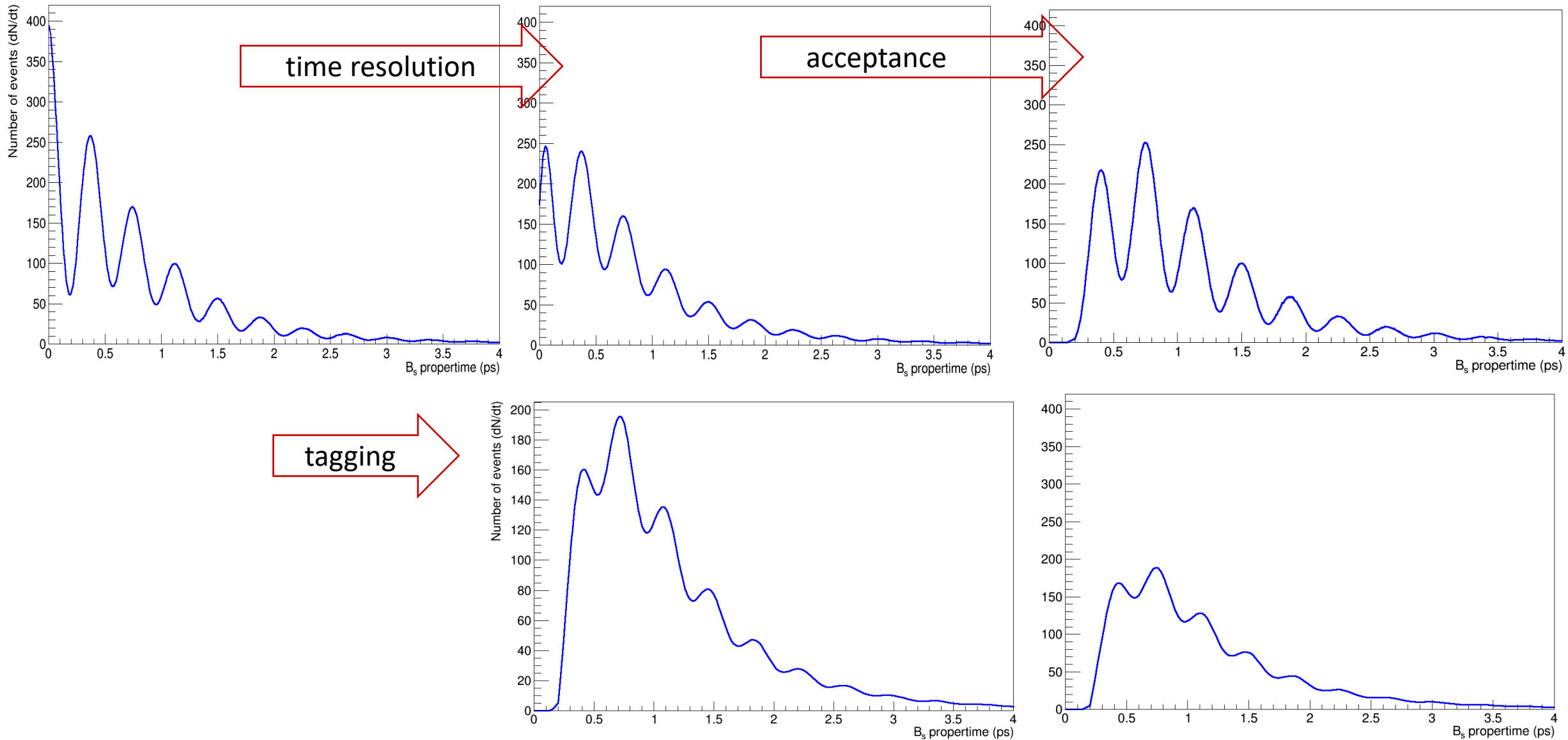
These relations should lead to the distribution like this:



$$A_{CP}(t) = \frac{\Gamma\{B(t) \rightarrow f\} - \Gamma\{\bar{B}(t) \rightarrow \bar{f}\}}{\Gamma\{B(t) \rightarrow f\} + \Gamma\{\bar{B}(t) \rightarrow \bar{f}\}}$$

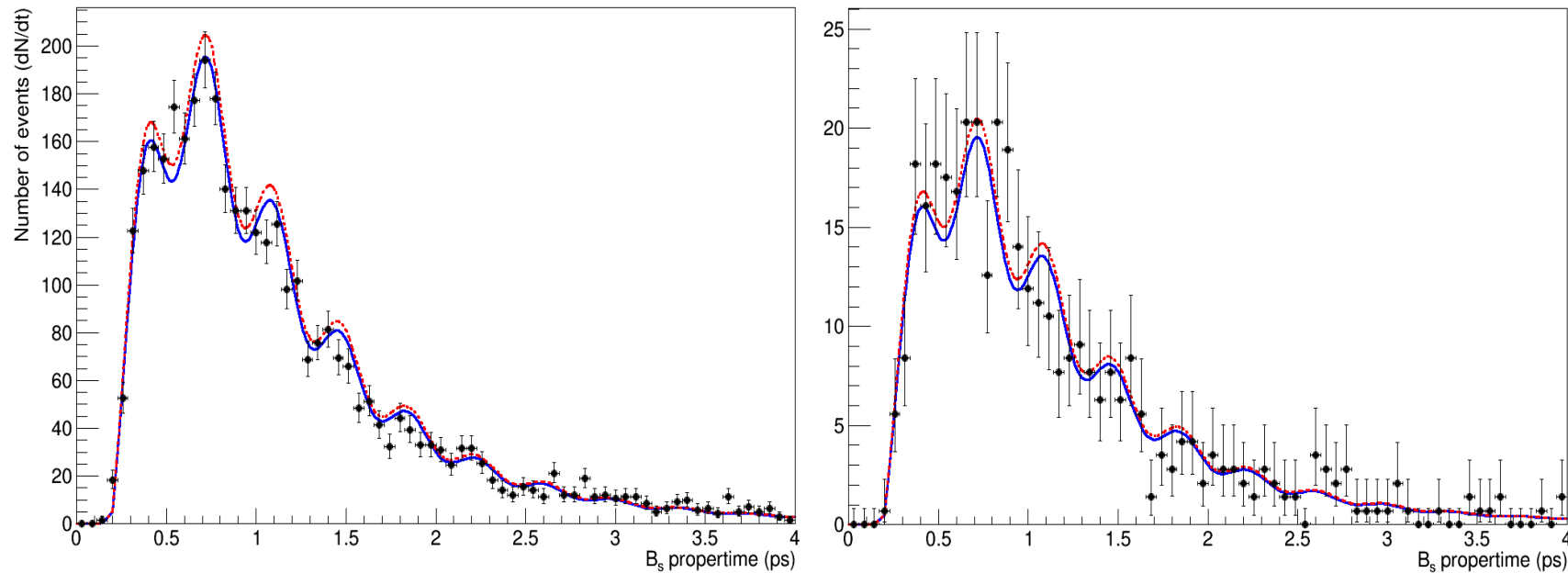
... but various detector effects have a major impact on time dependent decay rates:

# Time dependent $B_s^0 \rightarrow D_s^- K$ detector effects



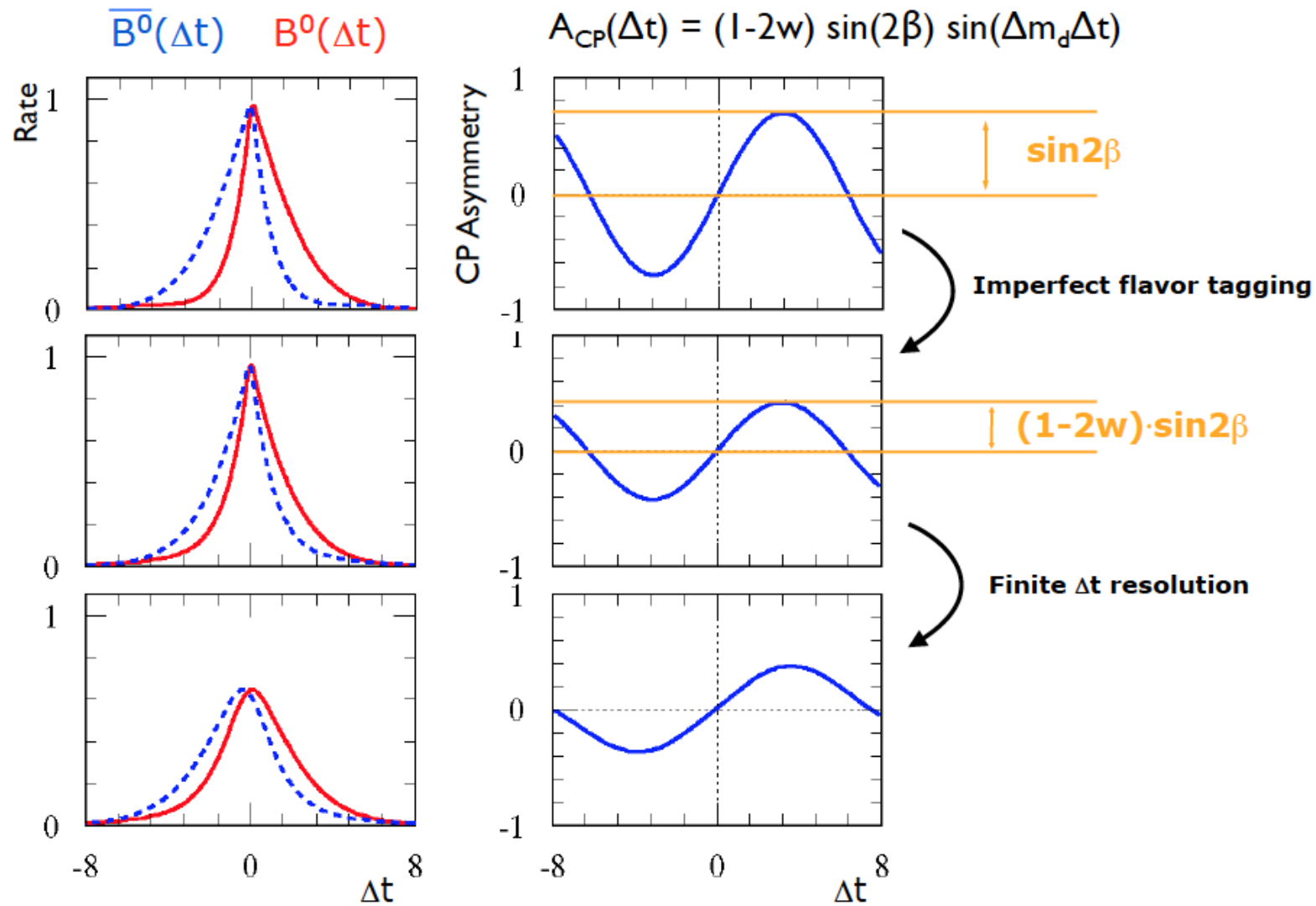
# Time dependent $B_s^0 \rightarrow D_s^- K$ detector effects

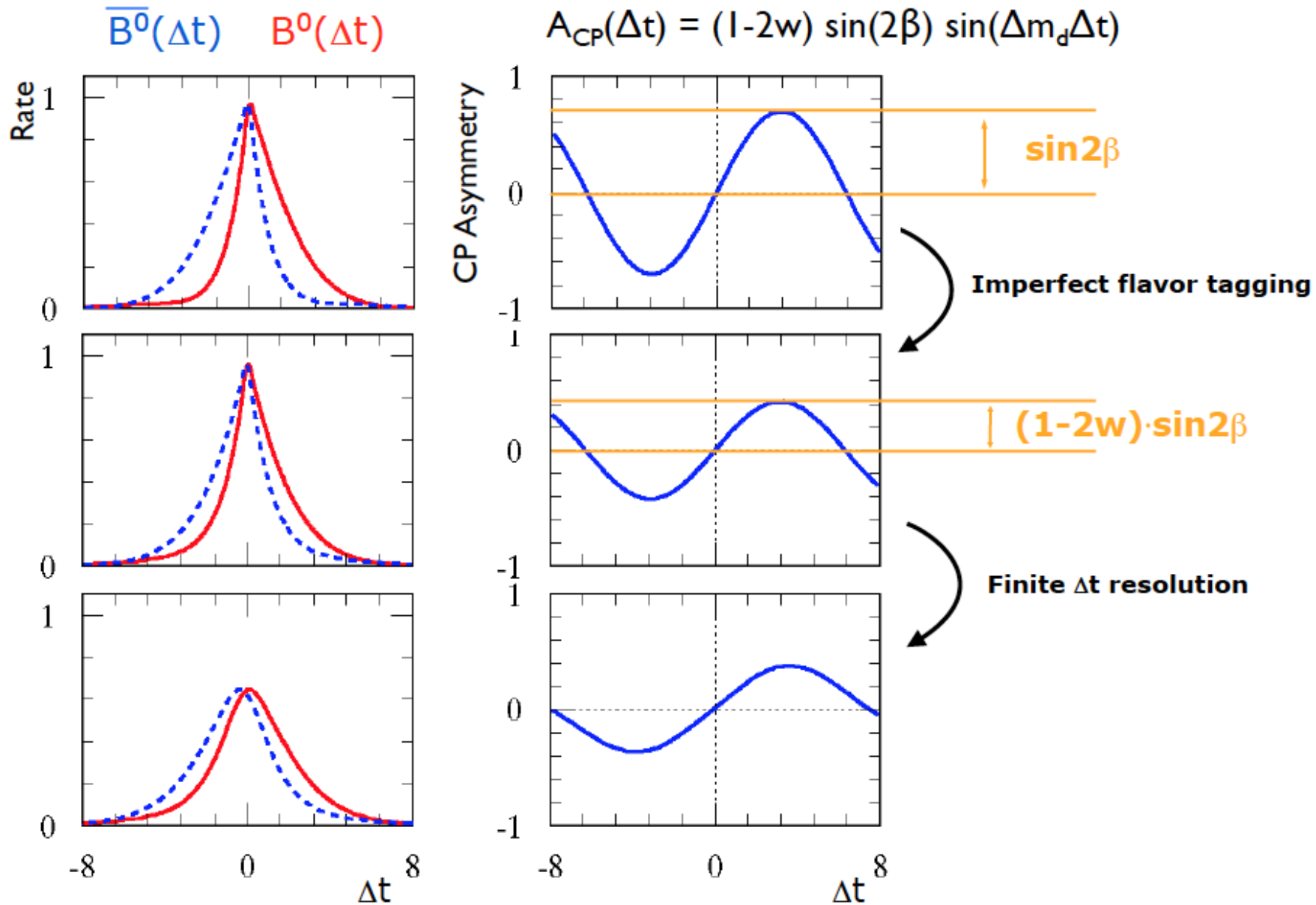
Roofit simulation of 10 years of LHCb data taking for this process....





# Mistag & dilution





# One practical remark

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When we write the equation for amplitudes and matrix elements we use standard Feynman rules:

- $V_{ud}$  when d-quark is incoming or anti-d is outgoing,
- $V_{ud}^*$  if incoming u quark or outgoing anti-u quark