CP Violation In Heavy Flavour Physics

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Short recap I (CP symmetry in K^0 mesons)

1. $K^0(\bar{s}d)$ and $\bar{K}^0(s\bar{d})$ are the eigenstates of the strong interactions but they are not eigenstates of the weak interactions.

$$\left.\begin{aligned} H_{Strong}|K^0\rangle &= m_{K^0}|K^0\rangle \\ H_{Strong}|\bar{K}^0\rangle &= m_{\bar{K}^0}|\bar{K}^0\rangle \end{aligned}\right\} \quad m_{K^0} = m_{\bar{K}^0} \approx 498\ MeV$$

2. The linear combinations of K^0 and $\bar{K}^0$:

$$\left.\begin{aligned} |K_1^0\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \\ |K_2^0\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \end{aligned}\right\}$$

K_1^0 and K_2^0 are eigenstates of $\mathcal{CP}$ operator

3. Kaons are produced as eigenstates of strong Hamiltonian but propagate through time as eigenstates of weak one.
4. But - since $|K_2^0\rangle$ is a mixture of $|K^0\rangle$ and $|\bar{K}^0\rangle$ states, even starting from pure $|K^0\rangle$ (or $|\bar{K}^0\rangle$) state we end up with a mixture of states of different strangeness (this so-called „**flavour oscillations**”)

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_1^0\rangle + |K_2^0\rangle) \qquad |\bar{K}^0\rangle = -\frac{1}{\sqrt{2}}(|K_1^0\rangle - |K_2^0\rangle)$$

Short recap II (CP Violation in K^0 mesons)

1. If CP symmetry holds for weak interactions, then K_1^0 must decay to two pions and K_2^0 must decay to three pions.
2. You are not surprised to know that a decay $K_2^0 \rightarrow \pi^0 + \pi^0$ and $K_2^0 \rightarrow \pi^+ + \pi^-$ has been observed.
3. So a new combination:

$$|K_S^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}} \underbrace{\left(|K_1^0\rangle + \epsilon|K_2^0\rangle\right)} \qquad |K_L^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}} \underbrace{\left(|K_2^0\rangle + \epsilon|K_1^0\rangle\right)}$$

- K_1^0 and K_2^0 are eigenstates of $\mathcal{CP}$ operator, whereas K_S^0 and K_L^0 are not.
- K_S^0 and K_L^0 are states with definite lifetimes $\boldsymbol{\gamma(\Gamma)_{S,L}}$ and distinct mass $\boldsymbol{m(M)_{S,L}}$
- ϵ is a small number describing the degree of $\boldsymbol{\mathcal{CPV}}$, K_S^0 is almost K_1^0, K_L^0 is K_2^0.
- Only 0.2% of K_L^0 violates $\mathcal{CP}$! Very tiny effect!
- This type of $\boldsymbol{\mathcal{CPV}}$ is called „indirect”

4. Solving the Schrödinger time dependent equation for a two-body system, we could calculate the probability of flavour oscillation phenomena.

5. **Now**: are there any other systems that could show us:
   - a flavour oscillation,
   - $\boldsymbol{\mathcal{CPV}}$ in much higher degree?

Time evolution of neutral mesons

1. In the absence of mixing, meson $\boldsymbol{K^0}$ can decay into all, allowed by energy-momentum conservation, states.
2. The exponential decay law leads to the time dependence of the wave function:

$$\boldsymbol{|K^0(t)\rangle = |K^0\rangle\, e^{-\frac{\Gamma t}{2}}\, e^{-imt}}$$

time evolution of a stable state with mass m, $m = E$

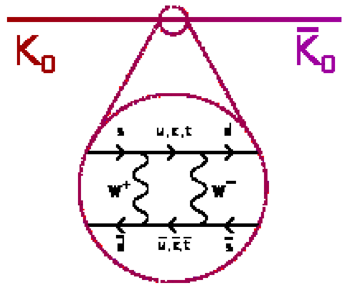
total width such that probability of finding an undecayed meson at time t is:

$$|\langle K^0(t)|K^0\rangle|^2 = e^{-\Gamma t}$$

which satisfy the equation:

$$i\frac{\partial}{\partial t}|K^0(t)\rangle = \underbrace{\left(m - \frac{i}{2}\Gamma\right)}_{H}|K^0(t)\rangle$$

3. If $\boldsymbol{K^0}$ can convert into $\boldsymbol{\overline{K^0}}$ through second order mixing diagram, the time evolution of a neutral meson must include both $\boldsymbol{K^0}$ and $\boldsymbol{\overline{K^0}}$:



$$\boldsymbol{|K^0(t)\rangle = e^{-iHt}|K^0(t=0)\rangle = e^{-iHt}\frac{1}{\sqrt{2}}\left(|K^0_S\rangle + |K^0_L\rangle\right) =}$$

$$= \frac{1}{\sqrt{2}}\left[e^{-i\left(m_S - \frac{i\Gamma_S}{2}\right)t}\,|K^0_S\rangle + e^{-i\left(m_L - \frac{i\Gamma_L}{2}\right)t}\,|K^0_L\rangle\right] = \ \dots = \ \dots = \ \dots$$

so let’s be more general:

$$\boxed{\boldsymbol{|\psi(t)\rangle = a(t)|K^0\rangle + b(t)|\overline{K^0}\rangle}}$$

Time evolution of neutral mesons

1. Assume that we have two neutral meson states: $\boldsymbol{P^0}$ and $\overline{\boldsymbol{P^0}}$ (they can be K^0, D^0, B^0) with an internal quantum number F such that $\Delta F = 0$ for strong and ELM interaction, but $\Delta F \neq 0$ for weak interactions.

$$|\Psi(t)\rangle = a(t)|P^0\rangle + a(t)|\bar{P}^0\rangle$$

only the coefficients are time-dependent, states- are not

2. The state obeys Schrödinger equation with total (effective) Hamiltonian (compare with the previous lecture):

$$i\frac{d}{dt}\binom{a}{b} = \mathcal{H}_{eff}\binom{a}{b} \equiv \left(M - \frac{i}{2}\Gamma\right)\binom{a}{b}$$

time evolution of coefficients only,
H is not hermitian,
$H = H_0 + H_W$

3. M and Γ are 2×2 Hermitian matrices (mass and decay)

$$\mathcal{H}_{eff} \equiv \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

we need to solve the eigenvalue problem for this system:

- diagonalise H matrix,
- find eigenvectors and eigenvalues (try this!)

4. $\mathcal{CPT}$ Theorem imposes that mass and width of particle P^0 and antiparticle $\overline{P^0}$ are the same, so: $\boldsymbol{H_{11} = H_{22}}$

Time evolution of neutral mesons

5. First some commonly used definitions:

$\boldsymbol{E_{1,2}}$ are eigenvalue that we have after solving the characteristic equation (H matrix should be diagonal):

$$|H - E\ I| = 0$$

$$\left(M - \frac{i}{2}\Gamma - E\right)^2 = \left(M_{12} - \frac{i}{2}\Gamma_{12} - E\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^* - E\right)$$

$$E_1 = m_1 - \frac{i\Gamma_1}{2} = \left(M - \frac{\Delta m}{2}\right) - i\left(\Gamma - \frac{\Delta\Gamma}{2}\right)$$

$$E_2 = m_2 - \frac{i\Gamma_2}{2} = \left(M + \frac{\Delta m}{2}\right) - i\left(\Gamma + \frac{\Delta\Gamma}{2}\right)$$

$$m_{1,2} = M \pm \frac{\Delta m}{2}$$

$$\Delta m = m_2 - m_1$$

$$\Gamma_{1,2} = \Gamma \pm \frac{\Delta\Gamma}{2}$$

$$\Delta\Gamma = \Gamma_2 - \Gamma_1$$

and eigenvector equation of the form:

$$(H - E\mathbb{I})\begin{pmatrix} p \\ \pm q \end{pmatrix} = 0$$

gives the relations:

$$\frac{p}{q} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}$$

Time evolution of neutral mesons

7. The eigenstates of effective Hamiltonian written in the form:

$$|P_1\rangle = p|P^0\rangle + q|\overline{P^0}\rangle$$

$$|P_2\rangle = p|P^0\rangle - q|\overline{P^0}\rangle$$

(compare with K_S^0 and K_L^0 as a mixture of K^0 and $\overline{K^0}$)

p and q are complex numbers satisfying: $|p|^2 + |q|^2 = 1$ (for K_1^0 and K_2^0 : $p = q = \frac{1}{\sqrt{2}}$)

8. Solving Schrödinger equation we see time evolution of the eigenstates (tutorial):

$$|P_1(t)\rangle = |P_1\rangle\, e^{-i\left(m_1 - \frac{i\Gamma_1}{2}\right)t}$$

$$|P_2(t)\rangle = |P_2\rangle\, e^{-i\left(m_2 - \frac{i\Gamma_2}{2}\right)t}$$

These relations show that the original P^0 meson after some time can either convert to $\overline{P^0}$ or decay.

Time evolution of neutral mesons

9. Finally the time evolution of weak eigenstates as a combination of flavour eigenstates:

$$|P^0(t)\rangle = f_+(t)|P^0\rangle + \frac{q}{p} f_-(t)|\overline{P^0}\rangle$$

$$|\overline{P^0}(t)\rangle = f_+(t)|\overline{P^0}\rangle + \frac{p}{q} f_-(t)|P^0\rangle$$

$$f_\pm(t) = \frac{1}{2}\left[e^{-i(m_1-\frac{i}{2}\Gamma_1)t} \pm e^{-i(m_2-\frac{i}{2}\Gamma_2)t}\right]$$

solve this!

$$\boxed{|f_\pm(t)|^2 = \frac{1}{4}\left[e^{-i\Gamma_1 t} + e^{-i\Gamma_2 t} \pm 2e^{-\bar{\Gamma}t}\cos(\Delta m t)\right]}$$

$$\bar{\Gamma} = \frac{\Gamma_1 + \Gamma_2}{2}$$

interference term

10. The time evolution of mixing probabilities, i.e. the probability that having started the observation with a P^0 meson, after some time t we still have P^0 (or it has oscillated to $\overline{P^0}$):

$$P(P^0 \to P^0; \mathrm{t}) = |\langle P^0|P^0(t)\rangle|^2 = |f_+(t)|^2$$

$$P(P^0 \to \overline{P^0}; \mathrm{t}) = |\langle \overline{P^0}|P^0(t)\rangle|^2 = \left|\frac{q}{p} f_-(t)\right|^2$$

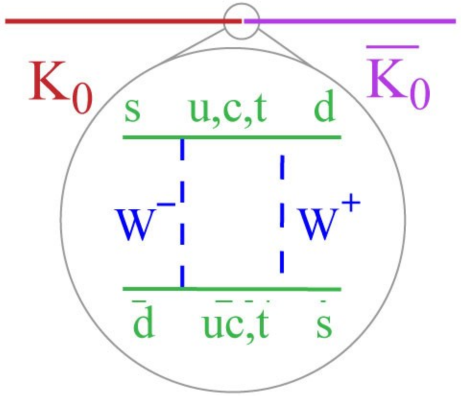
Let’s look closer at the parameters of flavour oscillations:

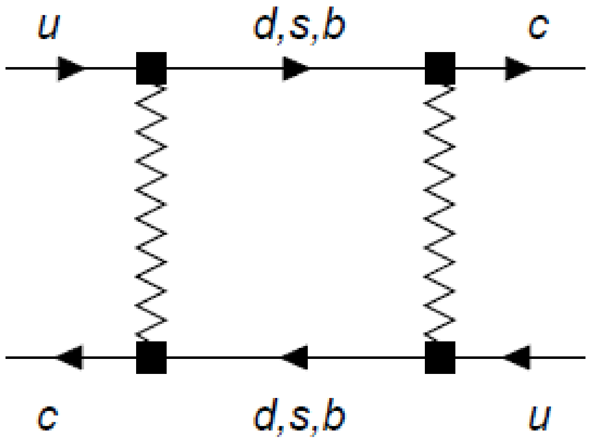
Time evolution of neutral mesons

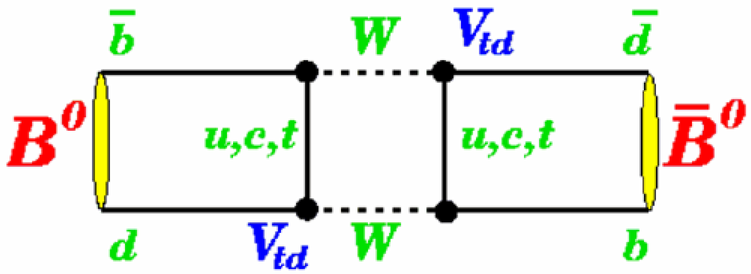
11. We can find four $\boldsymbol{P^0}$ -type mesons and investigate their flavour oscillations:

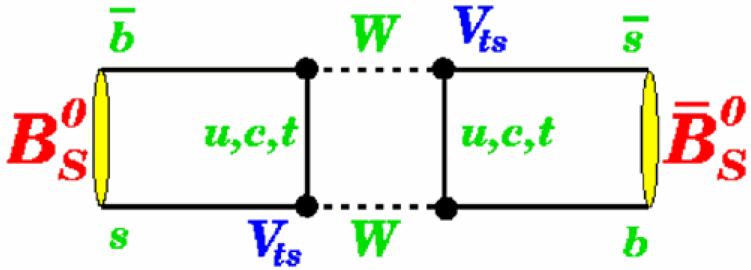
	d	s	b
$\bar{d}$	$\times$	K^0	B^0
$\bar{s}$	$\overline{K^0}$	$\times$	B_s
$\bar{b}$	$\overline{B^0}$	$\overline{B_s}$	$\times$

	u	c	t
$\bar{u}$	$\times$	D^0	$\diamond$
$\bar{c}$	$\overline{D^0}$	$\times$	$\diamond$
$\bar{t}$	$\diamond$	$\diamond$	$\times$



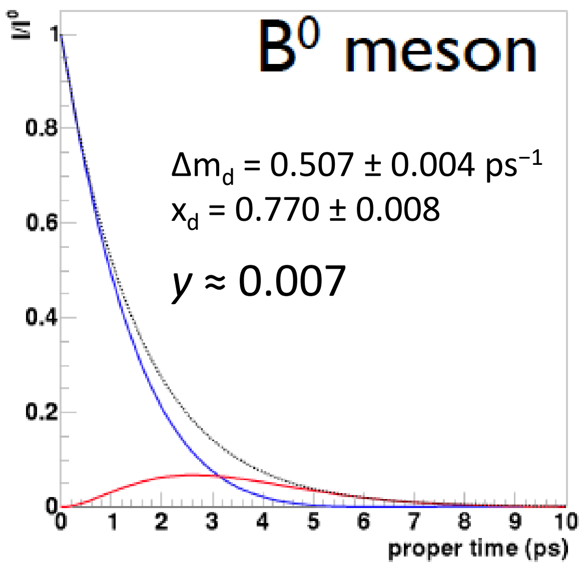


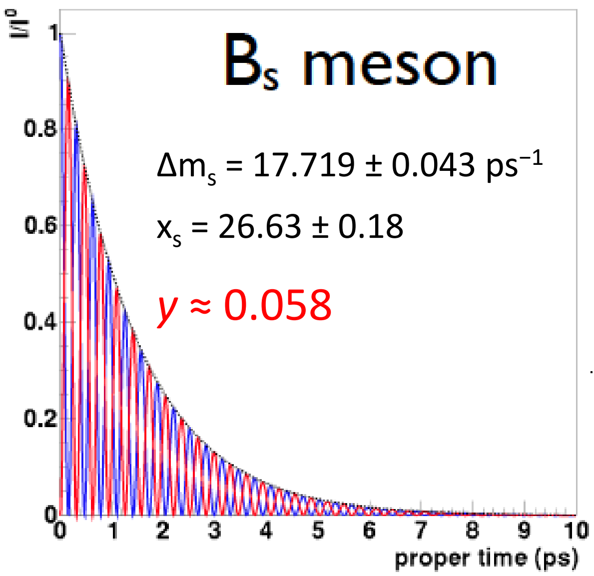




Two parameters determine period of oscillations compared to lifetime: $x = \dfrac{|\Delta m|}{\bar{\Gamma}}$ $\quad y = \dfrac{|\Delta\Gamma|}{2\Gamma}$

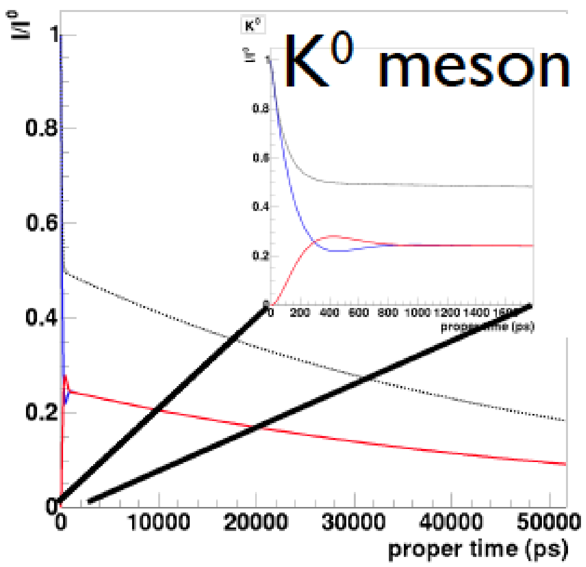
Time evolution of neutral mesons

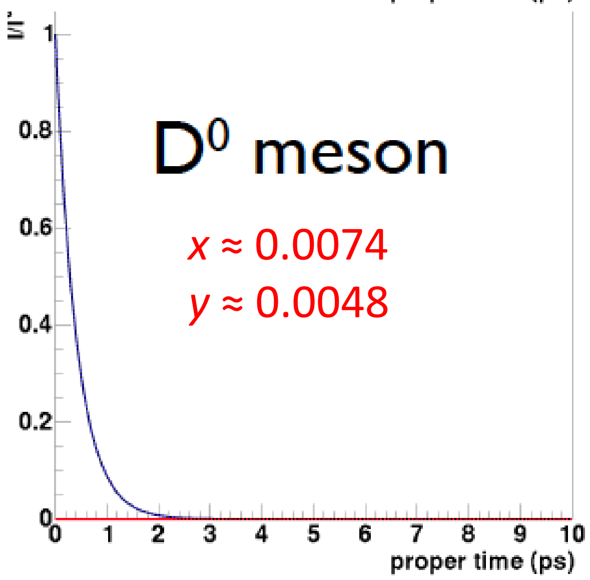




- D^0 mesons: *very, very* slowly
- K^0 mesons: very slowly
- B_d mesons: slowly
- B_s mesons: fast!

The frequency of B^0_s – anti-B^0_s oscillations is the highest.
On average, a B^0_s meson changes its flavour 9 times between production and decay





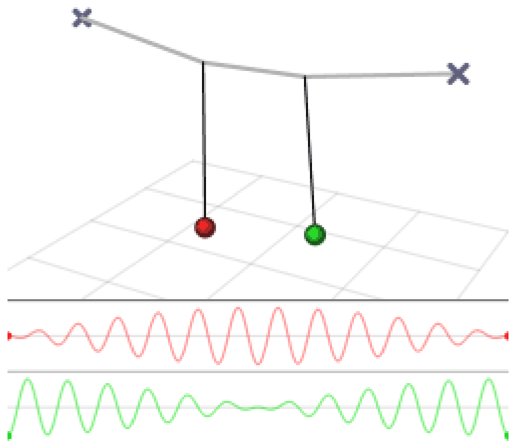
Blue:
given a P^0, at t=0, the probability of finding a P^0 at t.

Red:
given a P^0, at t=0, the probability of finding a P^0bar at t.

Project!

Summary of oscillations

1. Experimental puzzle with strange time life of a new particle has been solved by introducing weak decays of strange mesons.
2. Since they are produced in strong interactions and decay through weak, a second order box process occurs and is a way for flavour oscillations.
3. So neutral mesons behave like coupled damped pendulum.
4. This also holds for other neutral mesons.
5. In addition a CP violating decays were discovered in kaons decays.

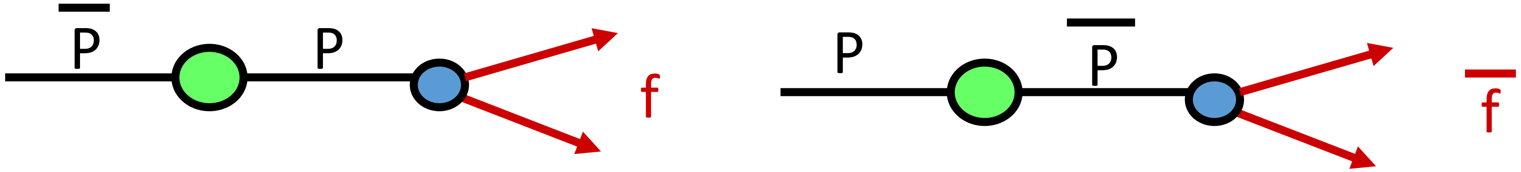


So now, the question is: how to connect mixing (flavour oscillations) with CP violation?

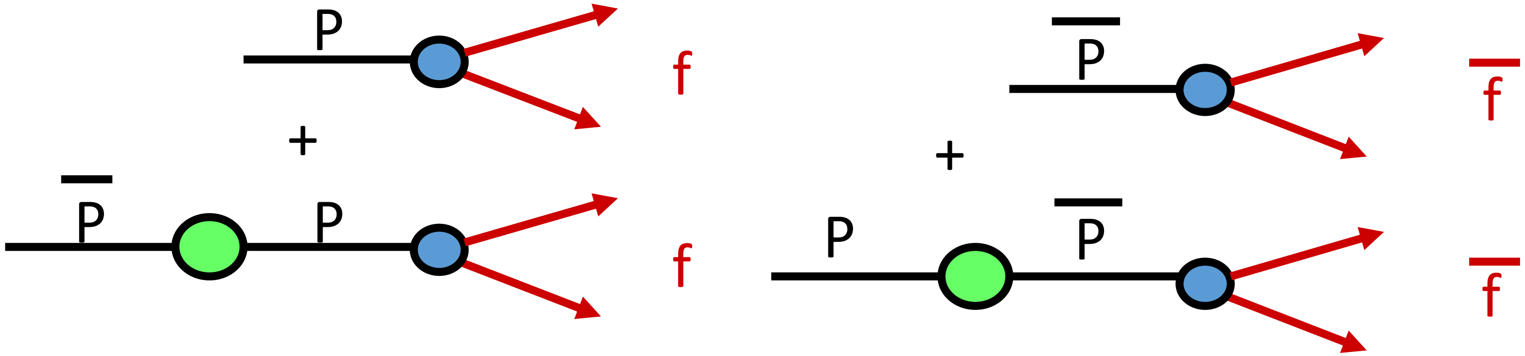
Types of CP Violation

I. CP violation in decay (direct CP Violation)

II. CP violation in mixing (indirect CP Violation)



III. CP violation in interference between mixing and decay



CP Violation in decay (direct)

1. One of the simplest way to discover $\mathcal{CPV}$ is to compare the decay rates $\Gamma(P \to f)$ with $\Gamma(\bar{P}) \to \bar{f}$
2. This is a method for direct $\mathcal{CPV}$ in decay amplitudes, when two amplitudes with different phases interfere.
3. If we define the asymmetry between $\mathcal{CP}$ conjugated decays, for charged and neutral mesons:

$$\boxed{A_{CP,dir} = \frac{\Gamma\{P \to f\} - \Gamma\{\bar{P} \to \bar{f}\}}{\Gamma\{P \to f\} + \Gamma\{\bar{P} \to \bar{f}\}}}$$

where: $\Gamma(P \to f) \propto |A_f|^2$

3. Amplitude A_f:

- is defined as a matrix element that describes the transition between state P and f, such that $P \to f$ depends on:

  $A_f = \langle f|H|P\rangle$ and $\bar{P} \to f$ on: $\overline{A_f} = \langle f|H|\bar{P}\rangle$

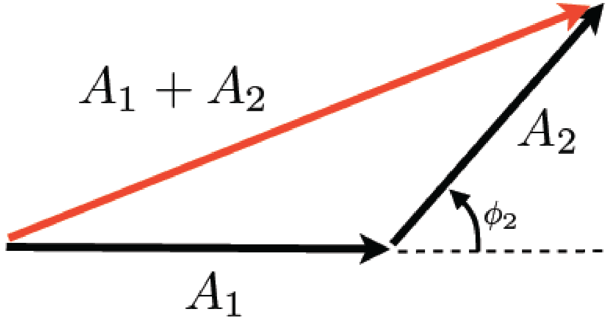
- is a complex number that can be written as a value A and phase: $A_f = A\, e^{i\phi}\, e^{i\delta}$
- Usually the amplitude A_f has a strong phase δ that is invariant under CP transformation and weak phase ϕ that changes sign under CP.

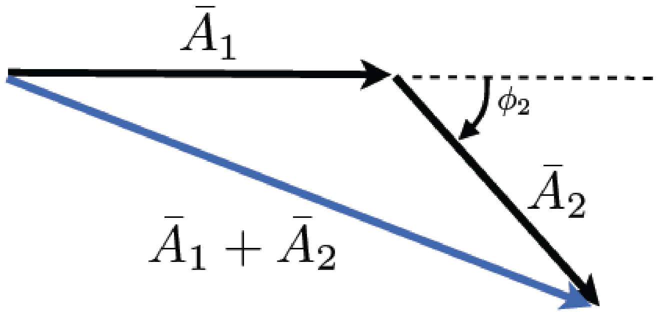
Essence of amplitude interference

$$A_j = \langle \text{final}|H_j|\text{initial}\rangle = |A_j| e^{+i\phi_j^{\text{weak}}}$$

CP

$$\bar{A}_j = A_j^* = |A_j| e^{-i\phi_j^{\text{weak}}}$$





$$P(i \to f) = |A_1 + A_2|^2$$

$$P(\bar{i} \to \bar{f}) = |\bar{A}_1 + \bar{A}_2|^2$$

$$= |A_1|^2 + 2|A_1||A_2|\cos\phi_2 + |A_2|^2$$

In case of only one decay amplitude – the decay rates are equal:

$$\boldsymbol{\Gamma(P \to f) = \Gamma(\bar{P} \to \bar{f})}$$

and no CP violation occurs.

For two amplitudes the decay rates may differ and the asymmetry is sensitive to relative phase

$$A = \frac{|\bar{A}_f|^2 - |A_f|^2}{|\bar{A}_f|^2 + |A_f|^2}$$

CP Violation in decay

4. Final state $\boldsymbol{f}$ can be $\mathcal{CP}$ eigenstate or not $\mathcal{CP}$ eigenstate. In the former additional amplitudes are written: $\overline{A}_{\bar{f}}$ and $A_{\bar{f}}$

5. The phase of the amplitude emerges only if we could find two different amplitudes that lead to the same final state, and:
   - their amplitudes had both different strong and weak phases,
   - then we would see evidence for direct $\mathcal{CP}$ violation (in decay) and decay rates will be different :

$$\boxed{\boldsymbol{\Gamma(P \to f) \neq \Gamma(\overline{P} \to \bar{f})}}$$

   - most general form of asymmetry:

$$A = \frac{\left|\overline{A_f}\right|^2 - \left|A_f\right|^2}{\left|\overline{A_f}\right|^2 + \left|A_f\right|^2} = \frac{2|A_1|\,|A_2| sin(\delta_1 - \delta_2)\, sin(\phi_1 - \phi_2)}{|A_1|^2 + |A_2|^2 + |A_1|\,|A_2|\, cos(\delta_1 - \delta_2)\, cos(\phi_1 - \phi_2)}$$

amplitude interference!

CP Violation in decay

6. We can also write a couple of asymmetries in a different form, e.g.:

$$A_f \equiv A(B^- \to f) = A_1\ e^{i\phi_1}\ e^{i\delta_1} + A_2\ e^{i\phi_2}\ e^{i\delta_2}$$

$$\bar{A}_{\bar{f}} \equiv \bar{A}\left(B^+ \to \bar{f}\right) = A_1\ e^{-i\phi_1}\ e^{i\delta_1} + A_2\ e^{-i\phi_2}\ e^{i\delta_2}$$

$$|A_f|^2 - |\bar{A}_{\bar{f}}|^2 = 2|A_1|\,|A_2| sin(\delta_1 - \delta_2)\ sin(\phi_1 - \phi_2)$$

$$\boxed{\boldsymbol{\Gamma(P \to f) \neq \Gamma(\overline{P} \to \bar{f})}}$$

- or (if there are more amplitudes leading to the state f) we can express this by:

show this!

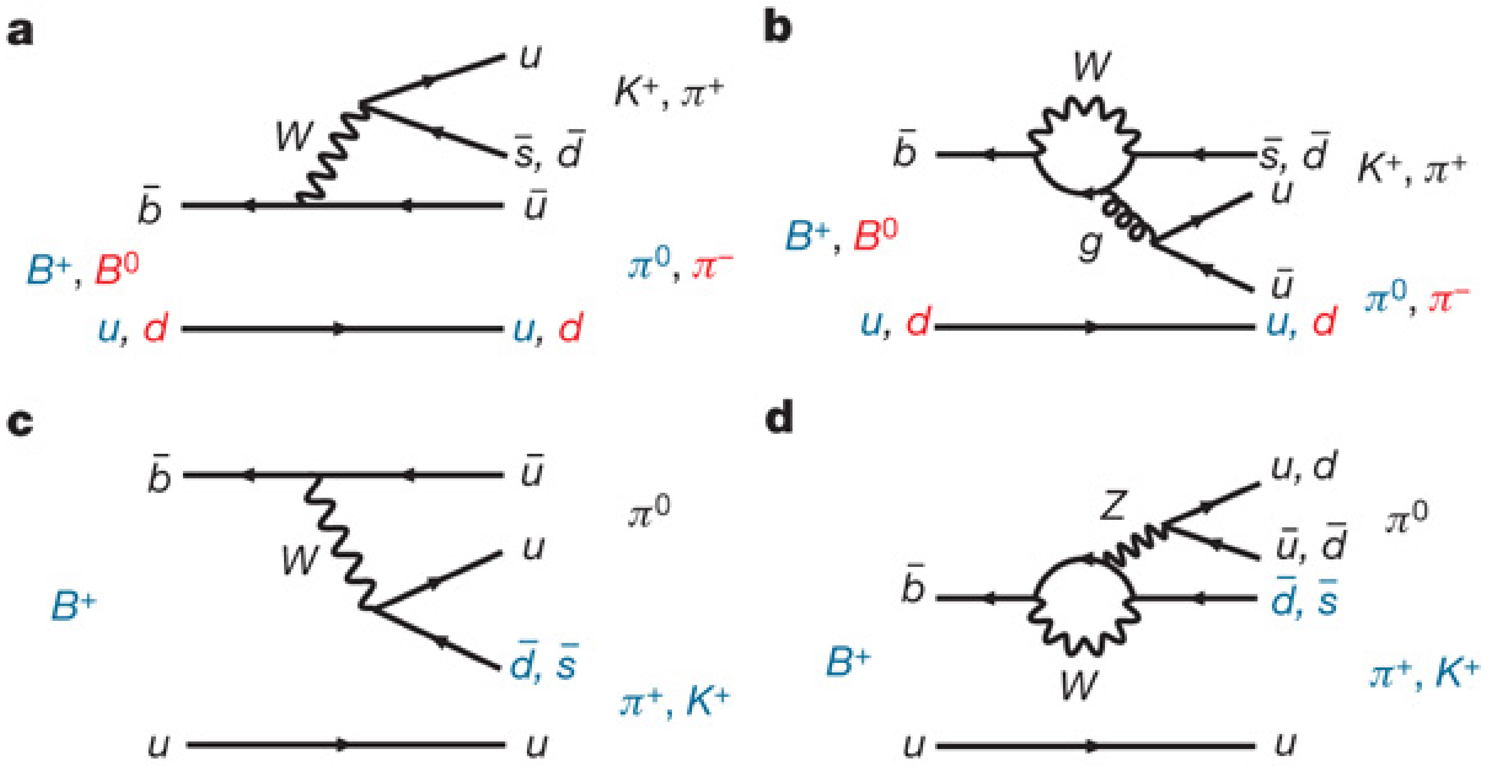
if $\mathcal{CP}$ is **NOT** conserved:

$$\boxed{\left|\frac{A_f}{A_{\bar{f}}}\right| = \left|\frac{\sum_i A_i e^{i\varphi_i} e^{i\delta_i}}{\sum_i A_i e^{-i\varphi_i} e^{i\delta_i}}\right| \neq 1}$$

if $\mathcal{CP}$ is conserved:

$$\left|\frac{\boldsymbol{A_f}}{\boldsymbol{A_f}}\right| = \boldsymbol{1}$$

CPV in decay



Think about experimental challenges!

- combinatorics,
- tagging,
- probability….

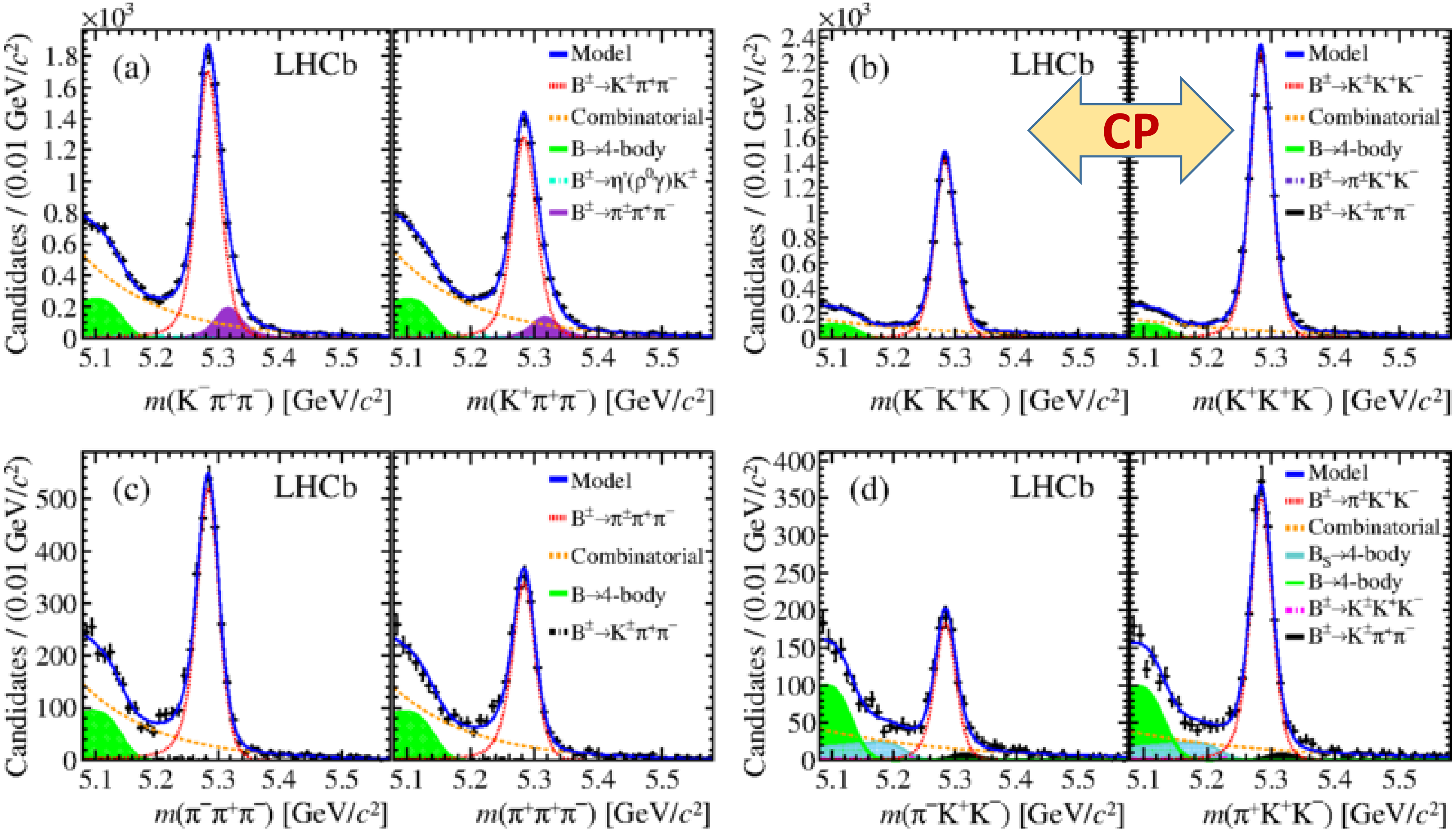
It is very common in flavour physics that simple ideas ($\mathcal{CPV}$ in differences in decay rates) are the most difficult for experiment.

CPV in decay

$B^+ \to K^- K^+ K^+$

Huge direct **CP** violation in decay amplitudes seen in B^-/B^+ decays

$B^- \to K^- K^+ K^-$



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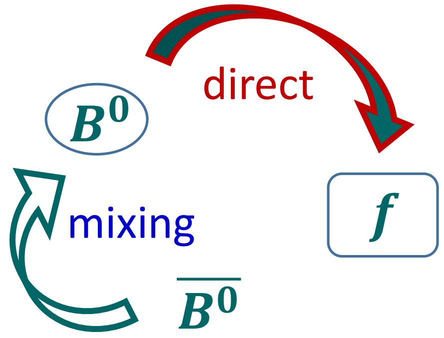
CP Violation in mixing

1. $\mathcal{CP}$ Violation in mixing (indirect $\mathcal{CP}$ violation) based on the spontaneous oscillations of a particle into antiparticle (mixing).
2. Mass eigenstates are different from $\mathcal{CP}$ eigenstates.
3. Only for neutral mesons! See kaons section.
4. Mixing does not necessarily mean $\mathcal{CP}$ violation, but can provide additional amplitude that can interfere.
5. Mixing rate for $\boldsymbol{P^0} \to \overline{\boldsymbol{P^0}}$ is different from $\overline{\boldsymbol{P^0}} \to \boldsymbol{P^0}$.
6. If the weak states of neutral meson are:

$$|P_1\rangle = p|P^0\rangle + q|\overline{P^0}\rangle$$

$$|P_2\rangle = p|P^0\rangle - q|\overline{P^0}\rangle$$



the $\mathcal{CP}$ symmetry is violated if:

$$\boxed{\left|\frac{q}{p}\right|^2 = \left|\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right| \neq 1}$$

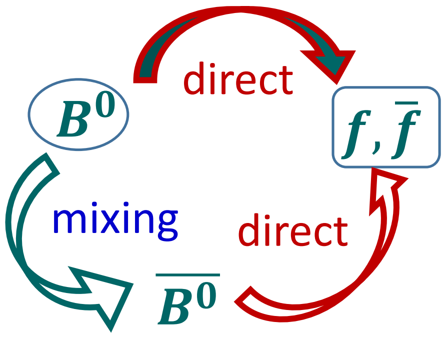
- very small effect,
- hard to measure because of hadronic uncertainties,
- so far no evidences of $\mathcal{CPV}$ in mixing

CP Violation in interference

1. *$\mathcal{CP}$* violation in interference between direct amplitude and amplitude after mixing.
2. Only for neutral mesons.
3. Some definitions:

$$A_f = \langle f|H|P^0\rangle \qquad \bar{A}_f = \langle f|H|\overline{P^0}\rangle \qquad \lambda \equiv \left(\frac{q}{p}\right)\left(\frac{\bar{A}_f}{A_f}\right) \qquad \bar{\lambda} \equiv \left(\frac{q}{p}\right)\left(\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}\right)$$

$$A_{\bar{f}} = \langle \bar{f}|H|P^0\rangle \qquad \bar{A}_{\bar{f}} = \langle \bar{f}|H|\overline{P^0}\rangle$$



4. *$\mathcal{CP}$* is conserved if: $\lambda = 1$ and $\bar{\lambda} = 1$
5. Try to calculate time dependent rates:

$$\boxed{\begin{aligned} \Gamma_f &= \left|\langle f|H|P^0(t)\rangle\right|^2 & \bar{\Gamma}_f &= \left|\langle f|H|\overline{P^0}(t)\rangle\right|^2 \\ \Gamma_{\bar{f}} &= \left|\langle \bar{f}|H|P^0(t)\rangle\right|^2 & \bar{\Gamma}_{\bar{f}} &= \left|\langle \bar{f}|H|\overline{P^0}(t)\rangle\right|^2 \end{aligned}}$$

homework!

$$|P^0(t)\rangle = f_+(t)|P^0\rangle + \frac{q}{p} f_-(t)|\overline{P^0}\rangle$$

6. And asymmetry (**time dependent!**)

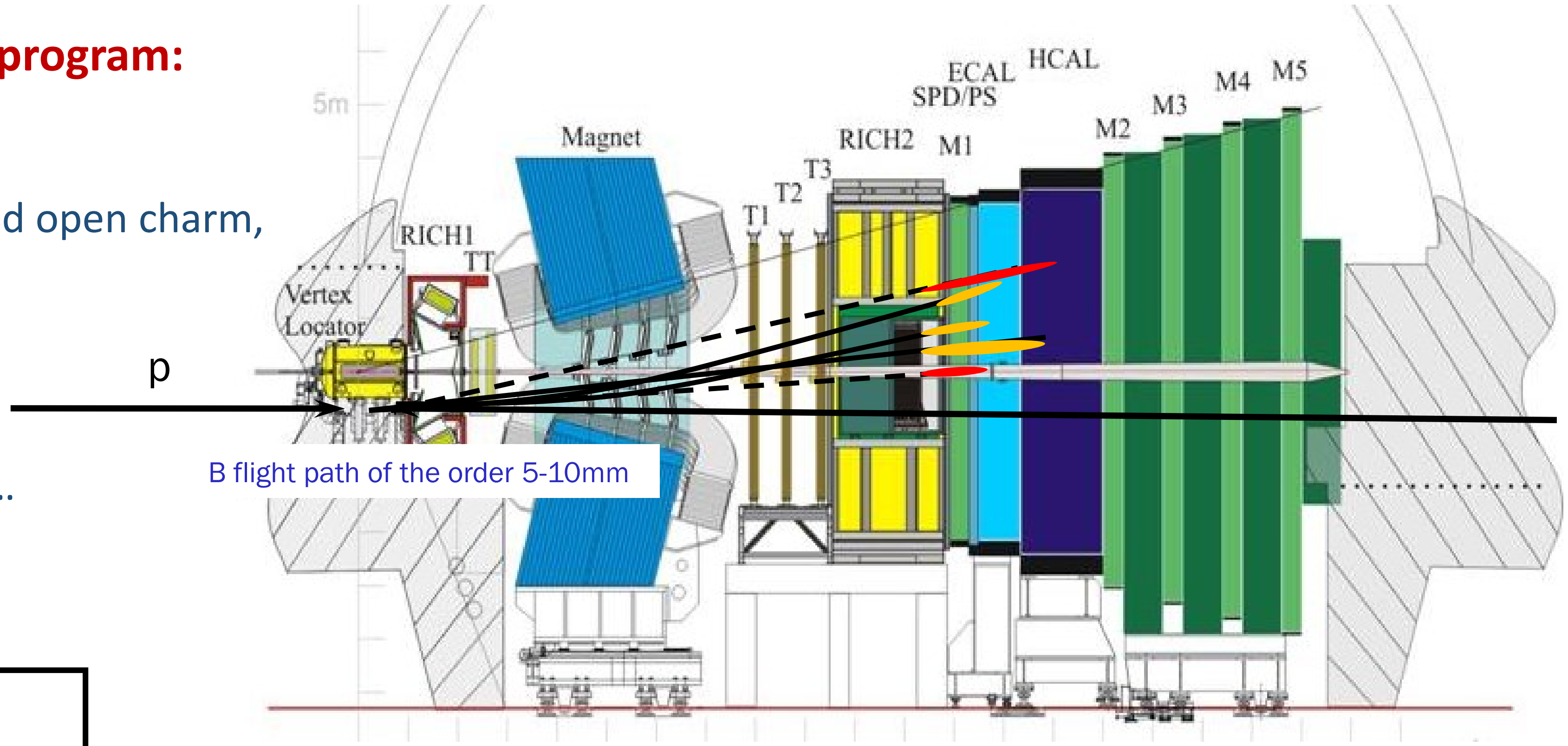
$$\boxed{A_{CP}(t) = \frac{\Gamma\{P(t) \to f\} - \Gamma\{\bar{P}(t) \to \bar{f}\}}{\Gamma\{P(t) \to f\} + \Gamma\{\bar{P}(t) \to \bar{f}\}}}$$

CPV – how to measure?

The experiment – LHCb spectrometer

Physics program:

- CP Violation ,
- Rare B decays,
- B decays to charmonium and open charm,
- Charmless B decays,
- Semileptonic B decays,
- Charm physics
- B hadron and quarkonia
- QCD, electroweak, exotica …



Tracking:
Silicon & Straw tubes
Magnetic field

Vertexing:
High precision silicon detectors (10μm position resolution) very close to collision point

RICH performance:
Cherenkov radiation.
Measures velocity, combine with momentum to get mass
Particle identification in p range 1-100 GeV
π, K ID efficiency > 90%, misID<~10%

Calorimeters:
Electromagnetic &
Hadronic calorimeters
- Critical (with muons) for triggering

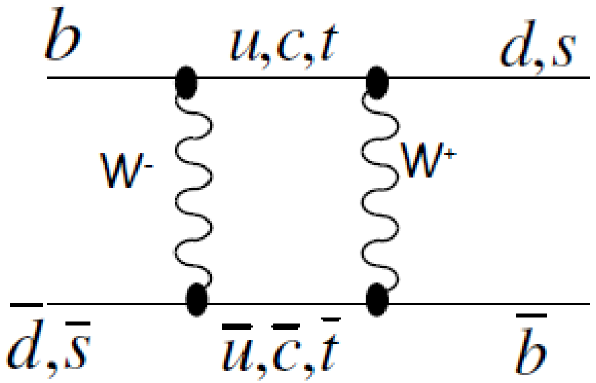
Mixing of B^0 and B_S^0 meson

1. Like neutral kaon system, neutral B mesons may also oscillate:
2. The top quark transition has the dominant amplitude:

$$A \propto \sum all\ pair\ of\ quarks\ A_{bi}A_{jb}^{*}$$

$$\begin{pmatrix} B^0 = d\bar{b} \\ \overline{B^0} = \bar{d}b \end{pmatrix}$$

$$\begin{pmatrix} B_S^0 = s\bar{b} \\ \overline{B_S^0} = \bar{d}s \end{pmatrix}$$



	$B^0 = d\bar{b}$ $\overline{B^0} = \bar{d}b$	$B_S^0 = s\bar{b}$ $\overline{B_S^0} = \bar{d}s$
Oscillations parameter	$x_d = \frac{\Delta m_d}{\overline{\Gamma_d}} \approx 0.72$	$x_s = \frac{\Delta m_s}{\overline{\Gamma_s}} \approx 24$
Large mass difference	$\Delta m_d \approx 3.3 \cdot 10^{-13}\ GeV \approx 0.5\ ps^{-1}$	$\Delta m_s \approx 17.8\ ps^{-1}$
Small lifetime difference	$x_d = \frac{\Delta\Gamma_d}{\overline{\Gamma_d}} \approx 5 \cdot 10^{-3}$	$x_d = \frac{\Delta\Gamma_s}{\overline{\Gamma_s}} \approx 0.1$
$\frac{q}{p}$ - sensitivity to weak phase	$\frac{q}{p} = \frac{V_{td}V_{tb}^{*}}{V_{tb}V_{td}^{*}} \sim \beta$	$\frac{q}{p} = \frac{V_{ts}V_{tb}^{*}}{V_{tb}V_{ts}^{*}} \sim \beta_s$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^{*}}{M_{12}}}$$

Mixing of B^0 and B_S^0 meson

1. The weak B-meson states are a combination of flavour states:

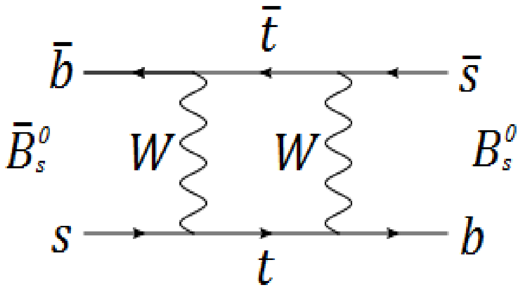
$$|B_L\rangle = p|B^0\rangle + q|\overline{B^0}\rangle \qquad\qquad |B_H\rangle = p|B^0\rangle - q|\overline{B^0}\rangle$$

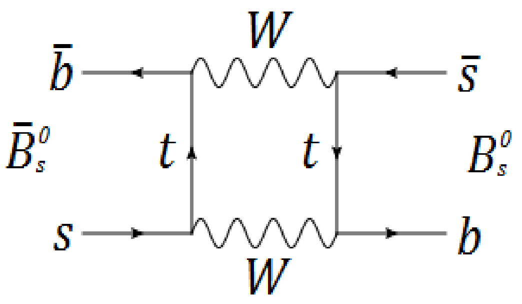
2. In terms of the CKM elements $\boldsymbol{q/p}$ is given by:

$$\frac{q}{p} = \frac{V_{td}V_{tb}^*}{V_{tb}V_{td}^*} = e^{-i2\beta}$$

here d is replaced by s in case of B_S^0

$$\frac{q}{p} = \frac{V_{ts}V_{tb}^*}{V_{tb}V_{ts}^*} = e^{-i2\beta_S}$$





so now the physical states are written as:

$$|B_L\rangle = 1/\sqrt{2}\left[|B^0\rangle + e^{-i2\beta}|\overline{B^0}\rangle\right]$$
$$|B_H\rangle = 1/\sqrt{2}\ \left[|B^0\rangle - e^{-i2\beta}|\overline{B^0}\rangle\right]$$

the eigenstates of the effective Hamiltonian, with definite mass and lifetime, are mixtures of the flavour eigenstates

and β is also called the $\boldsymbol{B^0}$ **mixing phase**

3. The states B_L and B_H are lighter and heavier state, with almost identical lifetimes: $\Gamma_L = \Gamma_H \equiv \Gamma$

4. The mass difference $\boldsymbol{\Delta m}$ between them is greater then in kaons.

Mixing of B^0 and B_S^0 meson

5. If we write the flavour states as a combination of weak states:

$$|B^0\rangle = 1/\sqrt{2}\,[|B_L\rangle + |B_H\rangle]$$

then the wavefunction evolves according to the time dependence of physical states:

$$\boldsymbol{|B(t)\rangle = 1/\sqrt{2}\{a(t)|B_L\rangle + b(t)|B_H\rangle\}}$$

where time dependence of coefficients is:

$$\boldsymbol{a(t)} = e^{-i(m_L-\frac{i}{2}\Gamma)t} \qquad \boldsymbol{b(t)} = e^{-i(m_H-\frac{i}{2}\Gamma)t}$$

Now substitute $a(t)$ and $b(t)$ and $|B_{L,H}\rangle$ into time-dependent wave function.

Do not forget to express mass states as a combination of flavour states….

$$|B_L\rangle = 1/\sqrt{2}\,[|B^0\rangle + e^{-i2\beta}|\overline{B^0}\rangle]$$
$$|B_H\rangle = 1/\sqrt{2}\,[|B^0\rangle - e^{-i2\beta}|\overline{B^0}\rangle]$$

Mixing of B^0 and B_S^0 meson

6. Now substitute $a(t)$ and $b(t)$ and $|B_{L,H}\rangle$ into time-dependent wave function:

$$|\boldsymbol{B(t)}\rangle = \boldsymbol{1/\sqrt{2}\{a(t)|B_L\rangle + b(t)|B_H\rangle\}}$$

$$|B_L\rangle = 1/\sqrt{2}\left[|B^0\rangle + e^{-i2\beta}|\overline{B^0}\rangle\right]$$

$$|B_H\rangle = 1/\sqrt{2}\ \left[|B^0\rangle - e^{-i2\beta}|\overline{B^0}\rangle\right]$$

$$\boxed{\boldsymbol{a(t)}} = e^{-i(m_L - \frac{i}{2}\Gamma)t} \qquad \boxed{\boldsymbol{b(t)}} = e^{-i(m_H - \frac{i}{2}\Gamma)t}$$

…. and calculate the probabilities of the state to stay as a $|B^0\rangle$

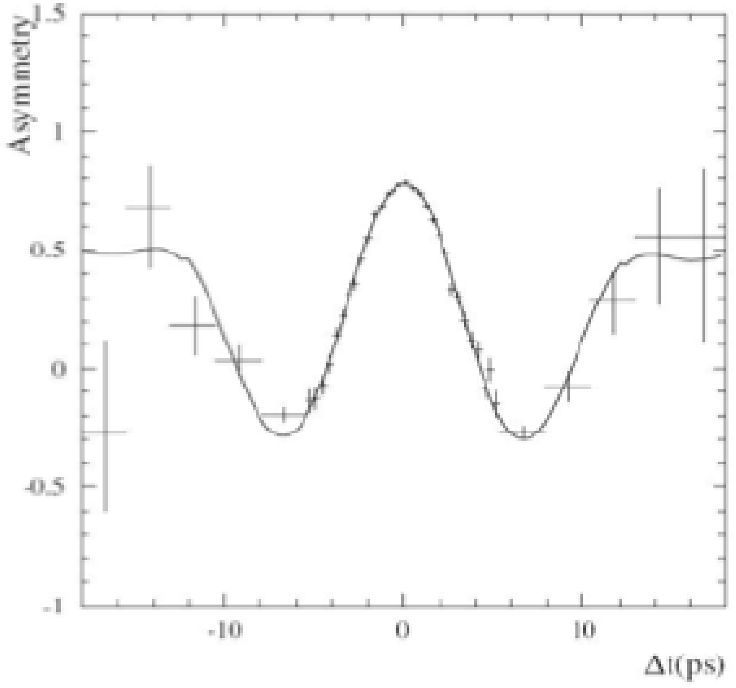
$$P(B^0(t=0) \rightarrow B^0; \mathrm{t}) = |\langle B^0(t)|B^0\rangle|^2 = .. = .. = \ e^{-\Gamma t}\,cos^2\left(\frac{\Delta m}{2}t\right)$$

7. The same calculation can be done for B_S^0

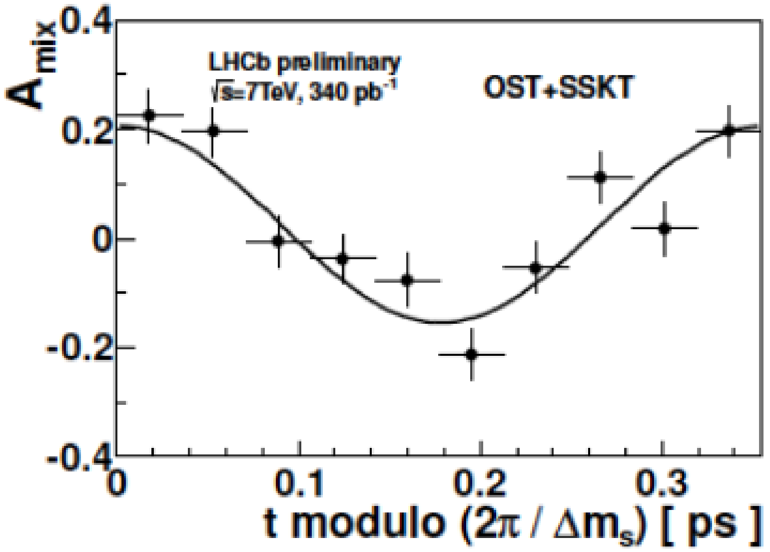
try to do it!

Mixing of B^0 and B_S^0 meson

BaBar: $\Delta m = 0.511 \pm 0.007\ ps^{-1}$



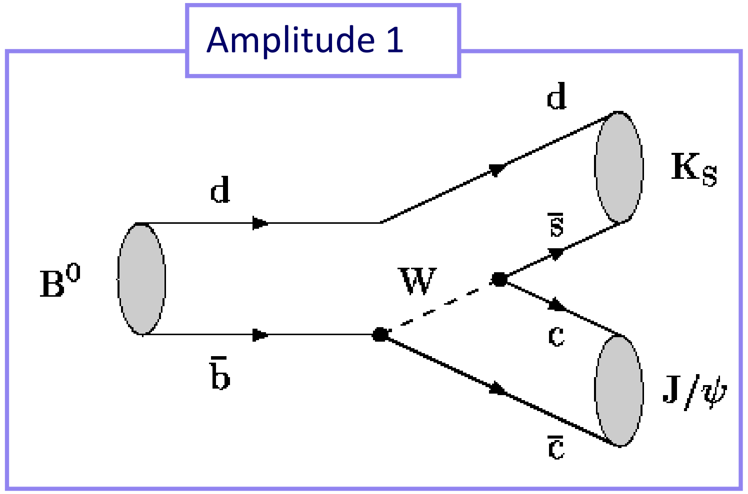
LHCb: $\Delta m_S = 17.768 \pm 0.023\ ps^{-1}$



$B_S^o \to D_S^- \pi^+$

Golden channel for $\sin 2\beta$

1. The process $\boldsymbol{B^0 \to J/\psi\ K_S}$ is called the „golden mode" for measurement of the β angle:
   a) clean theoretical description,
   b) clean experimental signature,
   c) large (for a B meson) branching fraction of order ~10^{-4}.
2. This is a process with interference of amplitudes with and without mixing:



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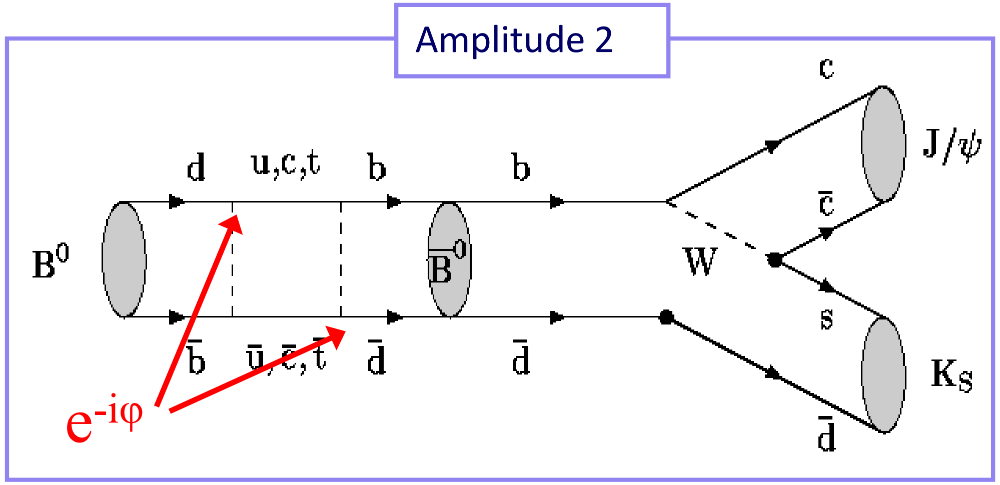


($\varphi=2\beta$)

3. The β angle sensitivity comes from the $\boldsymbol{B^0} \leftrightarrow \overline{\boldsymbol{B^0}}$ mixing due to the $\bar{t} \to \bar{d}$ and $t \to d$ transitions.

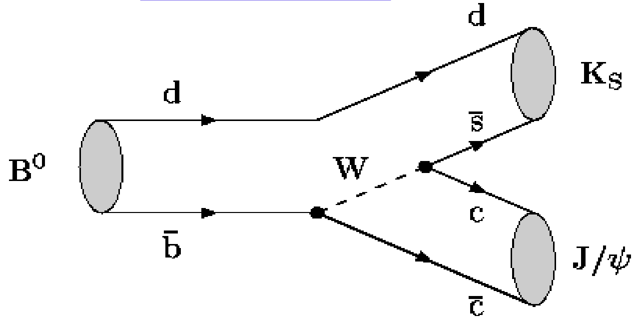
Golden channel for $\sin 2\beta$

4. We need to calculate the asymmetry of the type:

$$A_{CP}(t) = \frac{\Gamma_f - \overline{\Gamma_f}}{\Gamma_f + \overline{\Gamma_f}}$$

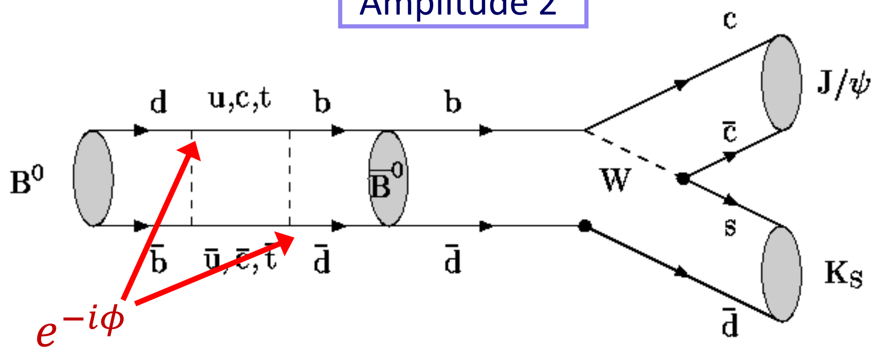
and remember that decay rate depends on (see lect 4): $\Gamma(B \to f) \propto |A_f|^2 = |A_1 + A_2|^2$

Amplitude 1



$+$

Amplitude 2



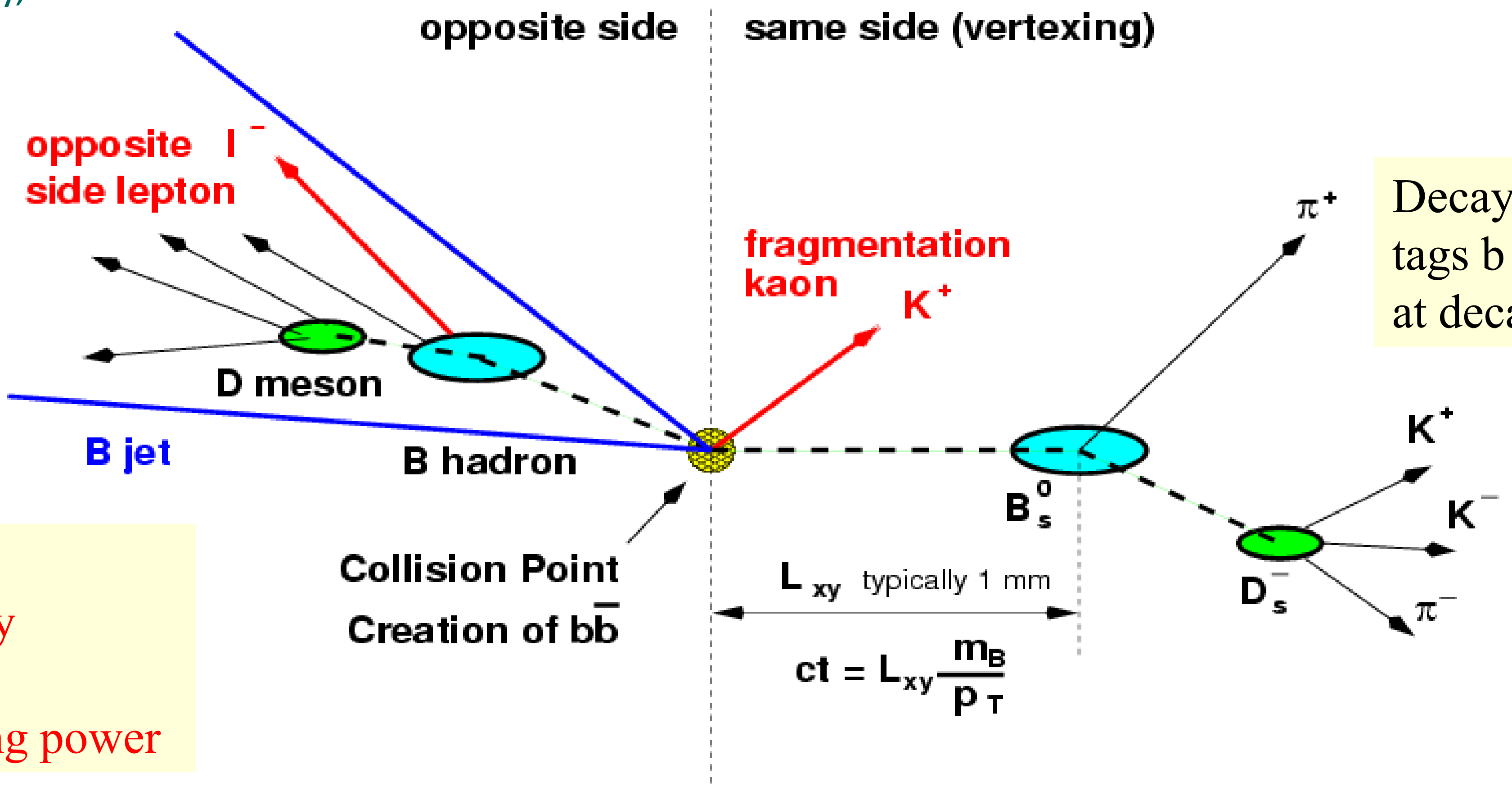
$\phi = 2\beta$

$$\Gamma(B \to J/\psi\ K_S) = \left| A e^{-imt-\Gamma t} \left(\cos\frac{\Delta mt}{2} + e^{-i\phi} \sin\frac{\Delta mt}{2} \right) \right|^2$$

$$A_{CP}(t) = \frac{\Gamma\{B \to J/\psi\ K_S\} - \Gamma\{\bar{B} \to J/\psi\ K_S\}}{\Gamma\{B \to J/\psi\ K_S\} + \Gamma\{\bar{B} \to J/\psi\ K_S\}} = -\sin 2\boldsymbol{\beta} \sin \Delta mt$$

Experimental challenges for mixing

1. Need to determine:
   a) Flavour at production ⇔ **tagging**
   b) Flavour at decay, from final state
   c) B decay length



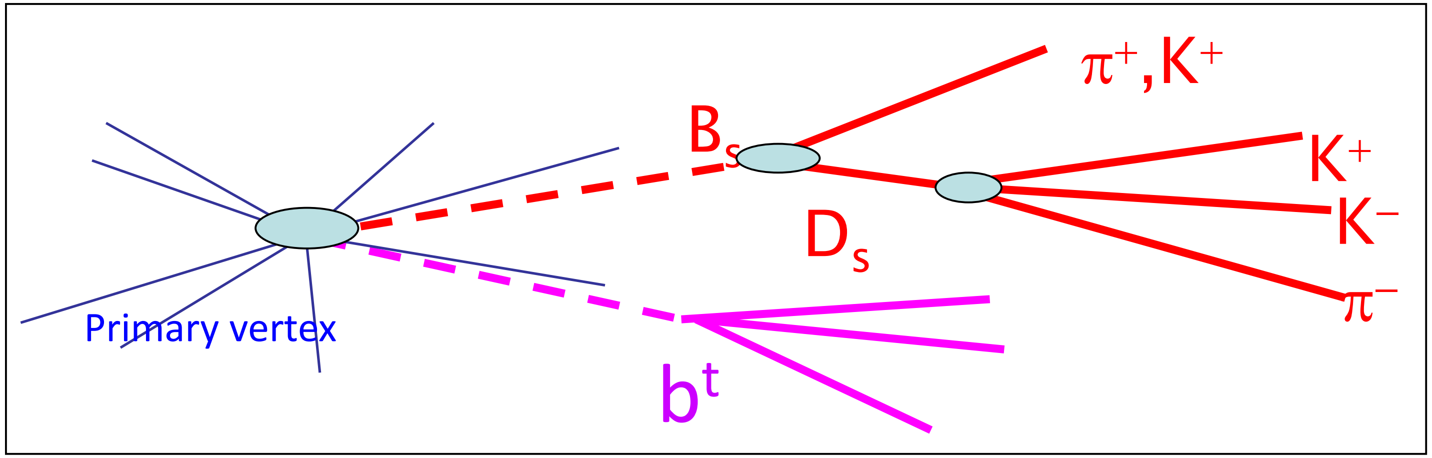
Decay mode tags b flavor at decay

Dilution $D = 1 - 2w$
w = mistag probability
ε = efficiency
εD^2 = effective tagging power

Time dependent $B_s^0 \to D_s^- K$

$B_s^0 \to D_s^- K$

This family of processes are very experimentally challenging:

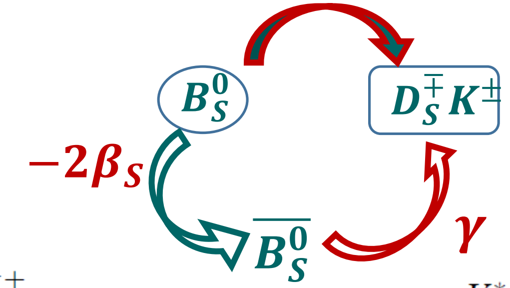


- six hadrons in the final state – very good PID and mass resolution
- high-P_T tracks and displaced vertices - *efficient trigger*
- *efficient tagging and good tagging power (small mistag rate)*
- *good decay-time resolution*

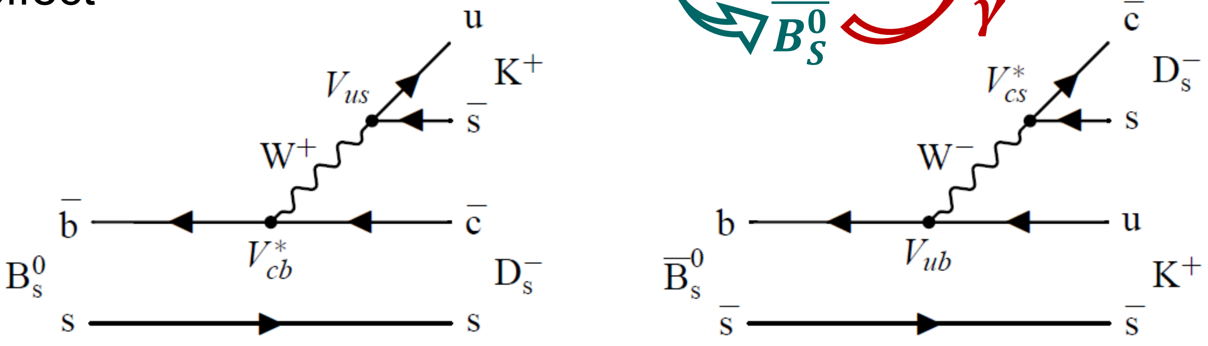
CP Violation in interference

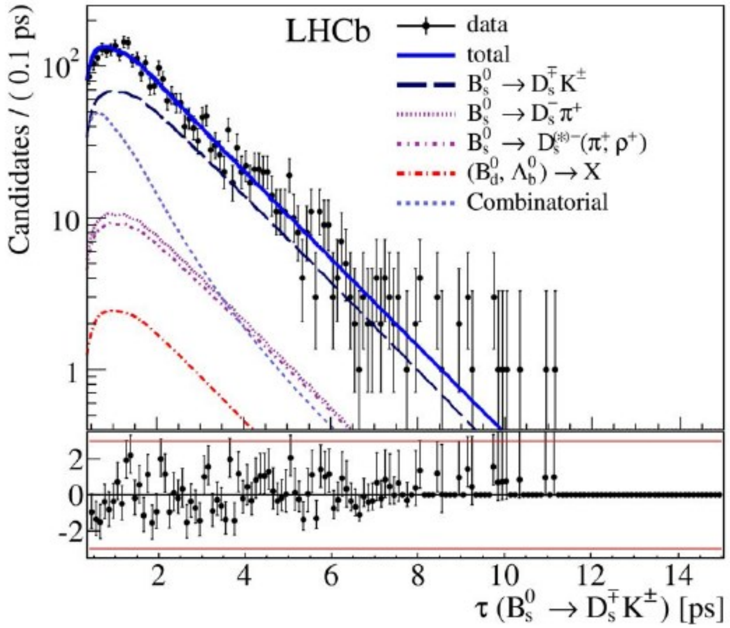
$$\boxed{B_S^0 \to D_S^{\mp(*)} K^{\pm(*)}}$$

Time dependent



1. Interference between mixing and direct decay, large effect because decays are not colour suppressed,
2. Sensitive to $(\gamma + \phi_s)$, strong phase δ,
3. Need to measure 4 time dependent decay rates





$$\Gamma_{B_S^0 \to f}(t)$$

$$= |A_f|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma_s t}}{2} \cdot \left(\cosh \frac{\Delta\Gamma_s t}{2} + \boldsymbol{D_f} \sinh \frac{\Delta\Gamma_s t}{2} + C_f \cos \Delta m_s t - \boldsymbol{S_f} \sin \Delta m_s t \right)$$

$$\boldsymbol{D_f \propto cos(\delta - (\gamma - 2\beta_S))}$$

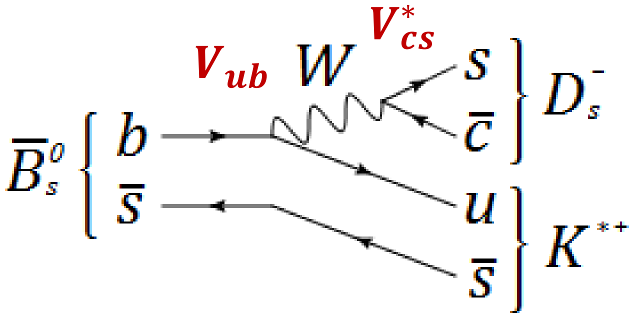
$$\boldsymbol{S_f \propto sin(\delta - (\gamma - 2\beta_S))}$$

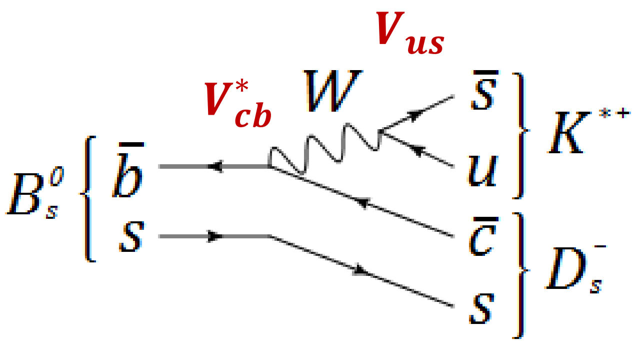
First measurement with this technique, 1fb^{-1}

Time dependent $B_s^0 \to D_s^- K$

1. B_s^0 and $\overline{B_s^0}$ decay to the same final state.

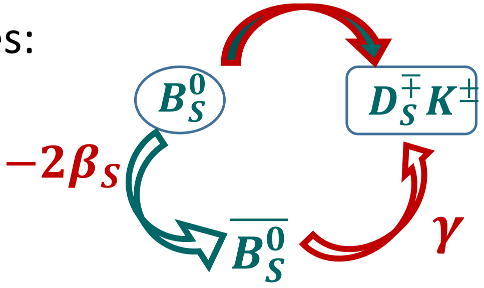
$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$





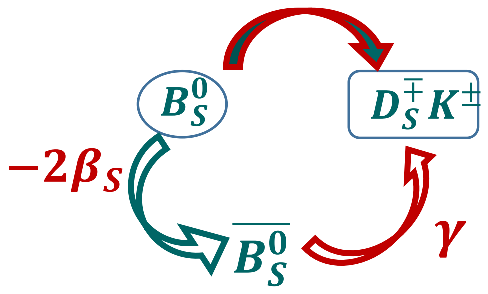
2. B_s^0 and $\overline{B_s^0}$can oscillate into one another.

3. So we have interference between two processes:



Time dependent $B_s^0 \to D_s^- K$

We have some experience in decay rate equation…
The probability of B meson decay to final state f is given by the Fermi golden rule:



$$\Gamma_{B_s^0 \to f}(t) \sim |\langle f|T|B_s^0(t)\rangle|^2$$

and we can try to calculate it…

$$\Gamma_{B_s^0 \to f}(t) = |A_f|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma_s t}}{2} \cdot \left(\cosh\frac{\Delta\Gamma_s t}{2} + D_f \sinh\frac{\boldsymbol{\Delta\Gamma_s} t}{2} + C_f \cos \boldsymbol{\Delta m_s} t - S_f \sin \Delta m_s t\right)$$

$$\Gamma_{\bar{B}_s^0 \to f}(t) = |A_f|^2 \left|\frac{p}{q}\right|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma_s t}}{2} \cdot \left(\cosh\frac{\Delta\Gamma_s t}{2} + D_f \sinh\frac{\Delta\Gamma_s t}{2} - C_f \cos \Delta m_s t + S_f \sin \Delta m_s t\right)$$

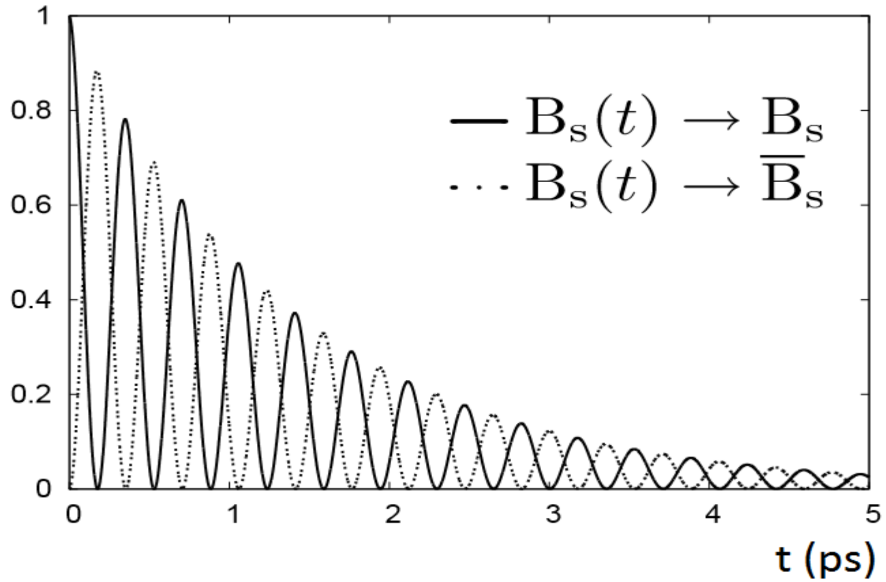
$$D_f = \frac{2Re\lambda_f}{1 + |\lambda_f|^2} \qquad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \qquad S_f = \frac{2Im\lambda_f}{1 + |\lambda_f|^2}$$

$$\lambda_f \equiv \frac{1}{\bar{\lambda}_f} = \frac{q}{p}\frac{\bar{A}_f}{A_f} \qquad A_f = \langle f|T|B_s^0\rangle \qquad \bar{A}_{\bar{f}} = \langle \bar{f}|T|\bar{B}_s^0\rangle$$

good luck!

Time dependent $B_s^0 \to D_s^- K$

These relations should lead to the distribution like this:

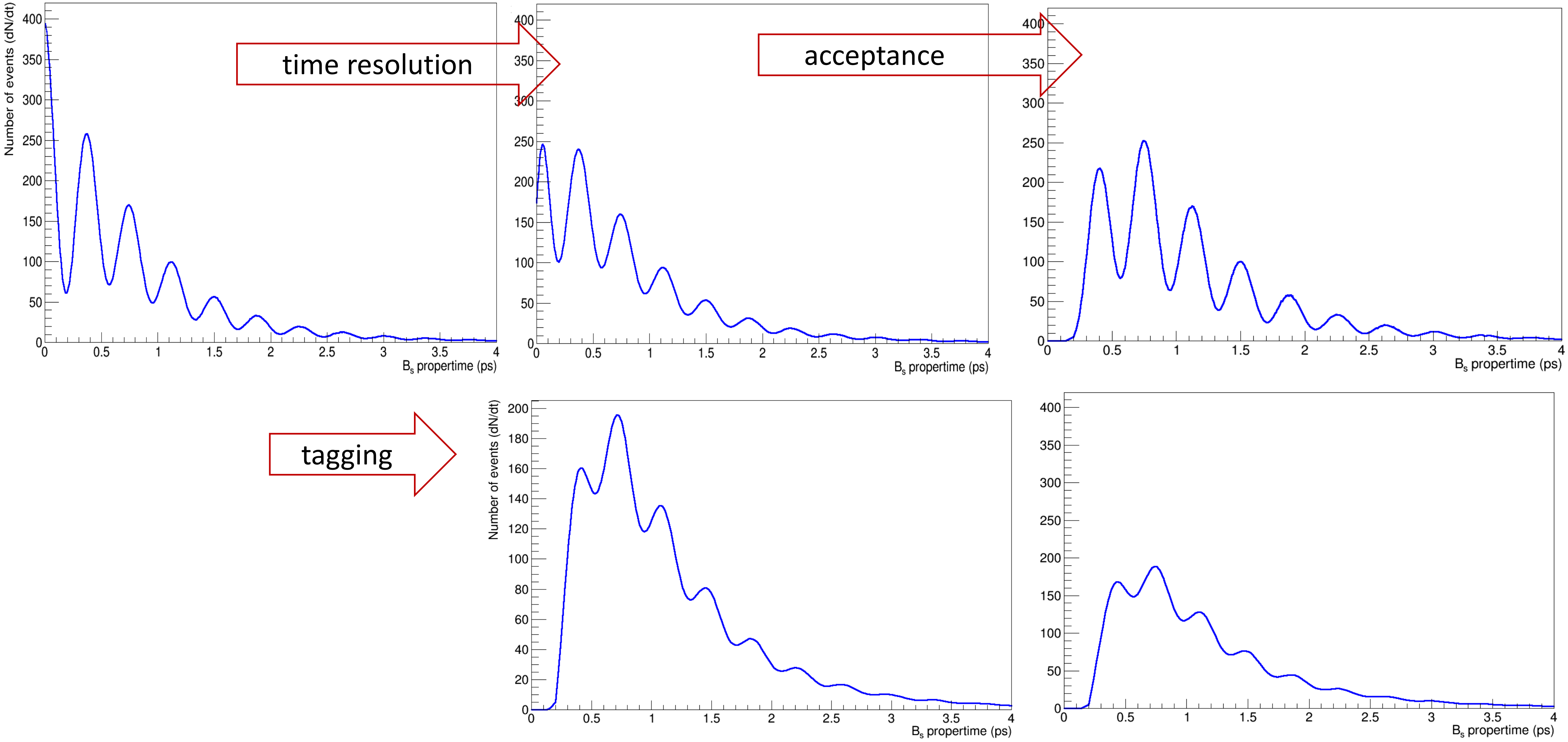


$$A_{CP}(t) = \frac{\Gamma\{B(t) \to f\} - \Gamma\{\overline{B}(t) \to \overline{f}\}}{\Gamma\{B(t) \to f\} + \Gamma\{\overline{B}(t) \to \overline{f}\}}$$

… but various detector effects have a major impact on time dependent decay rates:

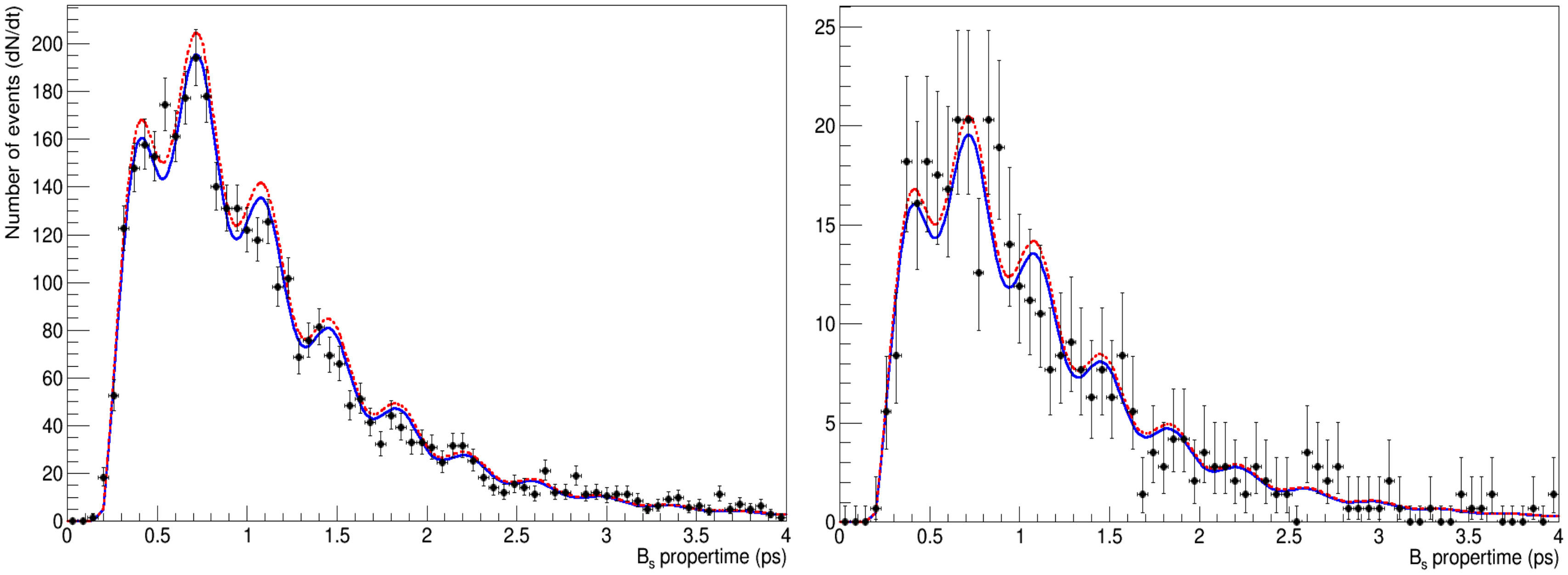
Time dependent $B_s^0 \to D_s^- K$ detector effects

Master Thesis of B.Bednarski

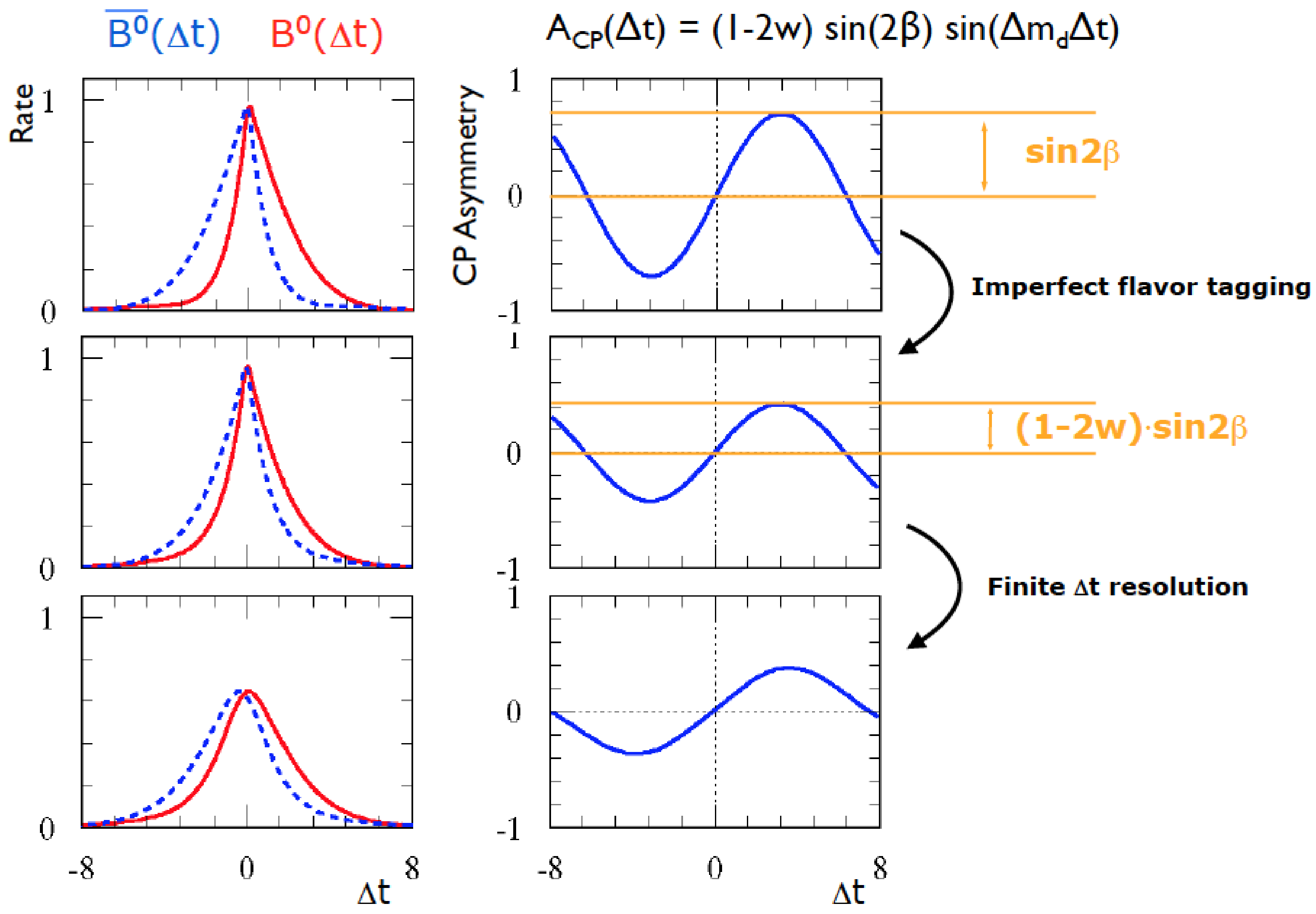


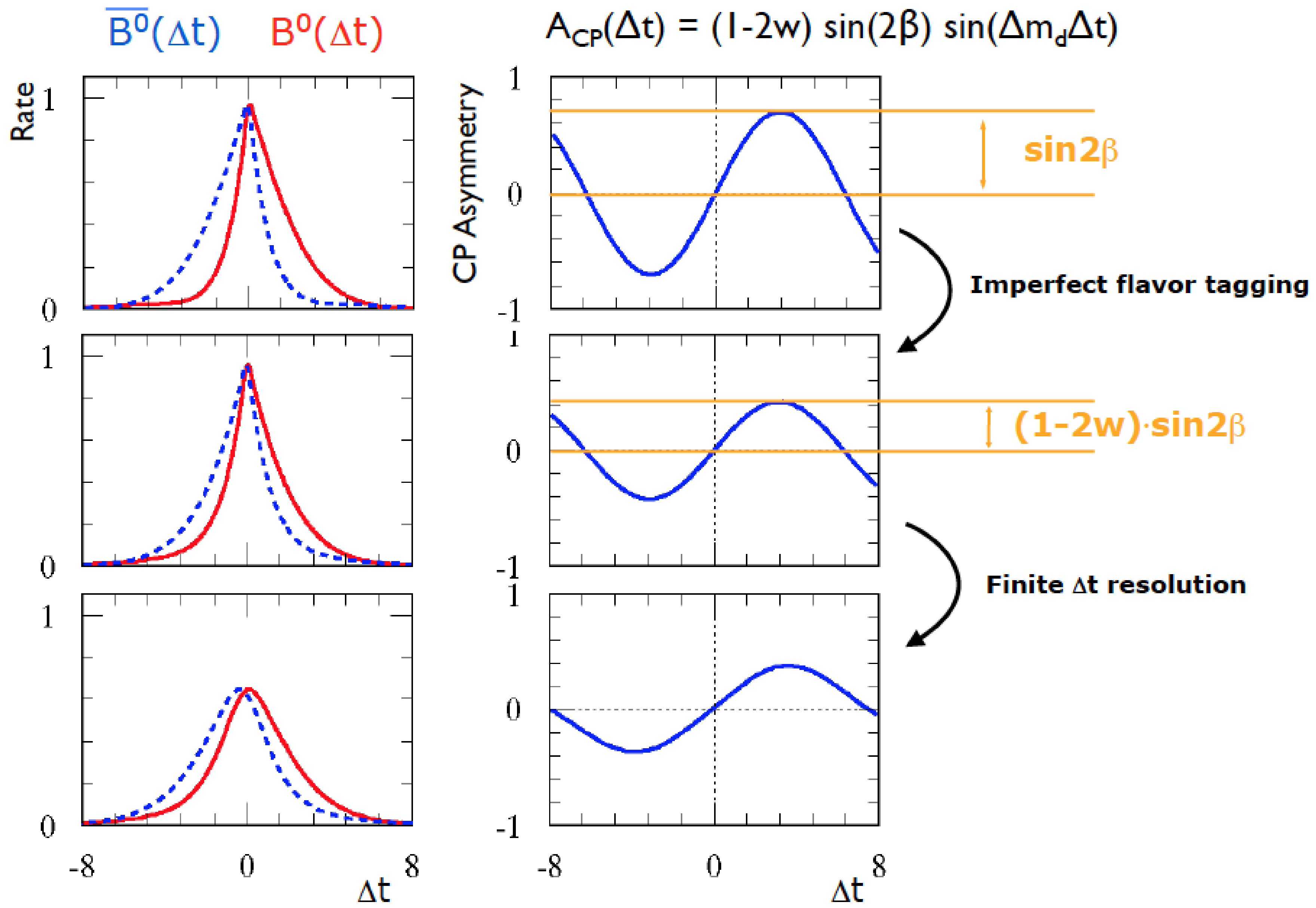
Time dependent $B_s^0 \to D_s^- K$ detector effects

Roofit simulation of 10 years of LHCb data taking for this process….



Mistag & dilution



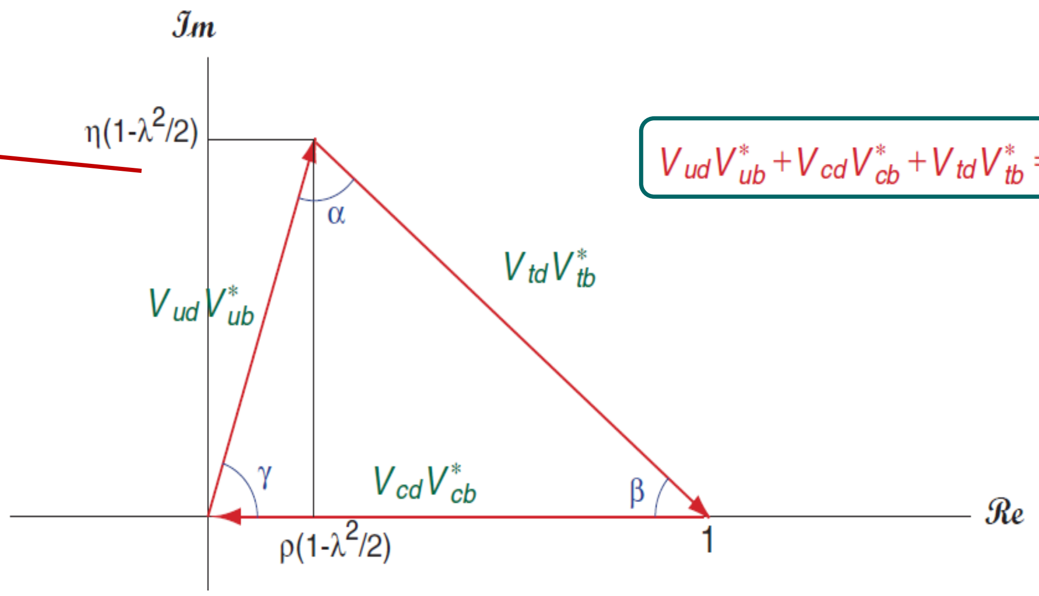

$\overline{B^0}(\Delta t)$ $B^0(\Delta t)$
$A_{CP}(\Delta t) = (1-2w)\sin(2\beta)\sin(\Delta m_d \Delta t)$
Rate
CP Asymmetry
$\sin 2\beta$
Imperfect flavor tagging
$(1-2w)\cdot\sin 2\beta$
Finite Δt resolution
-8
0
8
Δt
1
0
-1

One more thing…

The last subject to discus is CP Violation in the Standard Model (CKM matrix and Unitary Triangles)

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \boxed{\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta + i\eta\frac{\lambda^2}{2}) \\ -\lambda & 1-\frac{\lambda^2}{2} - i\eta A\lambda^4 & A\lambda^2(1+i\eta\lambda^2) \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



$$\boxed{V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0}$$